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Union oligopoly and entry in the presence of homogeneous labor

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Abstract. The analysis discusses the labor market equilibrium under union oligopoly, where unions represent homogeneous workers and use employment strategies. The following points are addressed: 1. The labor market outcomes in the presence of; a. uncooperative behavior among unions; b. uncooperative environment with a leading union; c. collusive (coordinated unions) behavior among unions; d. globally efficient bargaining, are confronted. A specific example with a Stone-Geary utility function and linear demand is forwarded. 2. Supply dynamics may push up employment and, therefore, the number of unions. In equilibrium, some bounds exist to the number of unions the market can support, which are investigated in the example. Five supply dynamics are considered: a. reservation wage restriction; b. a standard labor supply constraint; c. number of unions equal demand; d. individualistic unions; e. existence of a minimum (employed) membership requirement. The equilibrium number of unions for the Cournot-Nash, Stackelberg and efficient bargaining structures is derived for the case where unions exhibit Stone-Geary preferences and labor demand is linear.
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1. Introduction

In the two last decades, union strength and importance in wage determination seem to have decreased substantially; it is the main contribution of this article to offer a methodological understanding of the dynamics that may be behind such process. Hence, this research derives and compares the features of the labor market equilibrium in which several unions intervene under different strategic environments and follows to the "economics of union collapse" by allowing the number of unions in the economy to increase.

The multiple union setup has been modeled and its implications studied by authors as Oswald (1979) [2], Gylfason & Lindbeck (1984a) and (1984b)

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[2] Citing Rosen (1970) as the first author to recognize strategic interdependency among unions.

and others [3]. These studies usually assume heterogeneous labor with imperfect substitutability between workers and consider price, i.e., wage competition.

We are going to focus, instead, on homogeneous labor and consider that unions, as Hart´s (1982) syndicates, use employment strategies [4]. The use of these strategies is also a realistic assumption: legal restrictions to unemployment practices and laws restricting temporary labor contracts can be seen as part of the bargaining outcome; also, employment seems to be the concern of some strikes against firm bankruptcy or maintenance of partial contracts. This context has already been analyzed by the authors - Martins & Coimbra (1997) [5] - for the duopoly case. The present research is oriented towards the continuation of that research, and the extension of the study to the determination of the equilibrium number of unions the market may support.

We therefore start by considering the equilibrium conditions of four environments which differ with respect to the union and employer competitive (cooperative) behavior: Cournot-Nash strategies; Stackelberg equilibrium; efficient bargaining among the unions; efficient bargaining among unions and employers (which correspond to contract curve agreements in the one union case). This is presented in section II.

Section III is designed to derive more specific conclusions with respect to the labor market outcome through the use of Stone-Geary union preferences and, when necessary, a simple linear demand schedule. Symmetric solutions - i.e., in which unions' utility functions are alike - are also analyzed, allowing us to derive conclusions about the relation between the equilibrium wage and employment and the number of unions in the market.

In the presence of a fixed number of unions, labor demand determines equilibrium employment and wage. If there is unemployment, and a sort of closed-shop scenario in which employment - at least for that demand - can only be achieved through unionization, it is reasonable to conceive that unemployed (unionized and/or non-unionized) workers will form their own unions and push employment up - and/or wages down [6]. The supply pressure - that can be seen as due to "outsiders" [7] (in Lindbeck & Snower's (1988) sense) or unsatisfied "insiders" reaction when legally allowed - will

[3] Also Davidson (1988), Dixon (1988), Dowrick (1989), Jun (1989) and Dobson (1994), for example, where the effect of the existence of oligopoly in the product market is investigated.

[4] Notice that even if unions effectively consider price, i.e., wage, strategies, there will be equivalent quantity-employment strategies that will reproduce the former. See Martins & Coimbra (1997) for a justification. Also, Martins (1998) for the appraisal of dual reaction functions in the presence of heterogeneous labor.

[5] The reader is referred to the literature review there cited.

[6] As (positive) profit opportunities would attract new firms in the product market.

[7] We do not pursue a model of insider-outsider justification - as Solow (1985), for an example. Instead, we deal with an environment where "outsiders existence" is only justified by the closed shop agreement and/or some other legal requirement.

probably cause the number of unions to raise, at least till a certain point. This is the subject of section IV.

Different unions may be formed because labor force participants have different preferences over the employment-wage mix - therefore, membership assignment is a result of differentiated preferences of the labor force. But we do not pursue here a link to the membership dynamics literature. We are interested in union formation rather than union affiliation and argue that, as for firms in the product market model, the labor market may only support a fixed number of unions.

Notice that union entry is really a movement towards collapse of union effectiveness, even in a closed-shop scenario. If we consider industry-wide bargaining, union formation in most Western Countries is not usually restricted by labor legislation, apart from some representativeness requirements. Also, wage setting is commonly generalized to the whole industry or economy, which approaches the closed shop philosophy. Therefore, our study, even if theoretical, mirrors empirical realities.

The ways in which supply (or other) restricts union formation may be varied. We put forward five scenarios. Some of these are solutions with standard usage in the macroeconomics literature - the existence of a reservation wage, below which workers do not accept jobs, and a standard linear labor supply schedule. Others are suggested by the union scenario: aggregate employment goes up till the point where everybody who is unionized is (fully) employed - i.e., equilibrium number of unions equals demand; there is a fixed (exogenous) number of individuals in the economy which will form that many unions (in equilibrium they will/may not be fully employed). Finally, we consider another realistic situation: there is a legal requirement on the minimum number of employed members a union must exhibit to be considered representative in labor negotiations.

The solutions for Nash-Cournot unions, Stackelberg equilibrium and efficient bargaining unions are compared for symmetric unions with Stone-Geary utility functions and for a linear labor demand schedule.

The exposition ends with a summary of the main conclusions in section V.

2. Union oligopoly and other solutions [8].

Assume that there are n unions in the economy. The unions maximize the general utility function $U^i(L_i,W)$, increasing in the arguments and quasi-concave, for which U^i_L / U^i_W - the marginal rate of substitution between employment and wage - decrease with L_i and increases with W. Employment contracts are under closed-shop agreements, i.e., the firm(s) can only hire unionized workers. Demand is of the form:

[8] This section's results are a generalization of those presented in Martins & Coimbra (1997) for the two unions case, being, thus, presented in a sketchy manner.

$$\sum_{i=1}^{n} L_i = L(W) \tag{1}$$

or its inverse:

$$W = W(\sum_{i=1}^{n} L_i) \tag{2}$$

being negatively sloped, coming from maximization (in $L = \sum_{i=1}^{n} L_i$) of the (aggregate) profit function $\Pi(L,W) = P\ F(\sum_{i=1}^{n} L_i) - W \sum_{i=1}^{n} L_i$. Therefore, (2) establishes the value of the marginal product of labor, equal for all types of workers [9].

2.1. Cournot oligopoly

Each union maximizes

$$\underset{L_i,\, W}{\text{Max}}\ U^i(L_i, W) \tag{3}$$

$$\text{s.t.:}\quad \sum_{i=1}^{n} L_i = L(W) \quad \text{or} \quad W = W(\sum_{i=1}^{n} L_i) = P\,F_L(\sum_{i=1}^{n} L_i)\ , \quad \text{or}$$

$$\underset{L_i}{\text{Max}}\ U^i[L_i, W(\sum_{i=1}^{n} L_i)] \tag{4}$$

The optimal solution will obey

$$U^i_L / U^i_W = - W_L \quad \text{or} \quad L_i = R^i(L_1, \dots, L_{i-1}, L_{i+1}, \dots, L_n) = R^i(\sum_{j\neq i} S\ L_j) \tag{5}$$

$$i=1,2,\dots,n$$

where $R^i(\sum_{j\neq i} S\ L_j)$ denotes union i's reaction function, and labor demand [10].

[9] Nevertheless, most of the results below would also apply if this function represented the marginal revenue product of labor and if firms did not behave competitively in the product market.

[10] Existence of equilibrium is guaranteed by concavity of each union i's utility function with respect to L_i, and uniqueness is satisfied if $\Sigma_{i=1}^{n}\ dR^i/dL_{-i}\ /\ (1+dR^i/\ dL_{-i}) \leq 0$, - where

2.2. Union 1 is Stackelberg leader

The optimal solution will obey labor demand and

$$U^i_L + U^i_W W_L = 0 \quad \text{or} \quad L_i = R^i(L_1, \ldots, L_{i-1}, L_{i+1}, \ldots, L_n) \;\; i=2,\ldots,n \tag{6}$$

One can solve the system of n-1 equations in n unknowns in such a way that:

$$L_i = R'^i(L_1) \quad , \quad i=2, \ldots, n \tag{7}$$

Then, the leader, say 1, solves:

$$\underset{L_1, L_2, \ldots, L_n}{\text{Max}} \quad U^1[L_1, W(\sum_{i=1}^{n} L_i)] \tag{8}$$

$$\text{s.t.:} \quad U^i_L + U^i_W W_L = 0 \quad \text{or} \quad L_i = R^2(L_1), \; i = 2, 3, \ldots, n$$

Equilibrium is defined by (7), demand and:

$$U^1_L / U^1_W = - W_L \, (1 + \sum_{j=2}^{n} dR'^j / dL_1) \tag{9}$$

$$= (1 + \sum_{j=2}^{n} dR'^j / dL_1) \; U^i_L / U^i_W \; , \; i = 2, 3, \ldots, n$$

3.3. Efficient cooperation

Assume the unions cooperate with each other and we can extend the Nash-maximand approach to more than the 2 unions scenario. Then, unions maximize:

$$\underset{L_1, L_2, \ldots, L_n}{\text{Max}} \quad \prod_{i=1}^{n} \{U^i[L_i, W(\sum_{j=1}^{n} L_j)] - \bar{U}^i\}^{\delta_i} \tag{10}$$

δ_i is related to the strength of union i within the coalition, as justified by Svejnar (1986), extending the Nash-Zeuthen-Harsanyi solution. With $\delta_n = 1$, δ_i can be associated in "fair gambles" with M_i/M_n, where M_i denotes

$dR^i/dL_{-i} = dR^i/d(\Sigma_{j\neq i} S \; L_j)$ - which will hold if $-1 \leq dR^i / dL_{-i} \leq 0$, i=1,2,...,n. This ensures that optimal L_i falls as L rises. See Friedman (1983), p. 30-33.

number of members of union i. If we consider that $\sum_{i=1}^{n} \delta_i = 1$, then we can link $\delta_i = M_i / \sum_{j=1}^{n} M_j$ [11].

Eventually, (10) could represent the utility function of a unique union with workers with different preferences over the wage-employment mix, having n types of workers, with M_i workers of type i, i =1, 2,..., n. With perfect substitution between workers, ultimately the wage paid by the firm(s) must be the same for all workers.

F.O.C yield:

$$\delta_i U^i_L / [U^i(L_i, W) - \bar{U}^i] = - W_L \sum_{j=1}^{n} \delta_j U^j_W / [U^j(L_j, W) - \bar{U}^j] \qquad (11)$$

$i = 1, 2, ..., n$

Therefore, as the right-hand-side is the same for any i:

$$\delta_i U^i_L / [U^i(L_i, W) - \bar{U}^i] = \delta_j U^j_L / [U^j(L_j, W) - \bar{U}^j] \qquad (12)$$

This can be seen as a distribution (across unions) equation.

In this case, the equilibrium will obey labor demand and [12]:

$$\sum_{i=1}^{n} U^i_W / U^i_L = - L_W = - 1 / W_L \qquad (13)$$

As we have seen [13], this case reproduces the monopoly union behavior.

[11] See Martins & Coimbra (1997) for a justification of the relation between δ_i and number of members of union i.

[12] Efficiency conditions, in Edgeworth tradition, would also come from the solution of the problem

$$\max_{L_1, L_2, ..., L_n, W} U^1(L_1, W)$$

$$\text{s.t.:} \quad U^j(L_j, W) \geq \bar{U}^j \;, \quad j = 2, 3, ..., n$$

$$W = W\left(\sum_{i=1}^{n} L_i\right)$$

[13] Martins & Coimbra (1997).

2.4. Fully efficient bargaining

$$\underset{L_1,\dots,L_n,W}{\text{Max}} \prod_{i=1}^{n} [U^i(L_i, W) - \bar{U}^i]^{\delta_i} \; [\Pi [(\sum_{i=1}^{n} L_i), W] - \bar{P}] \tag{14}$$

δ_i represents the strength of union i relative to the employer side, hence, δ_i / δ_j represents the strength of union i relative to union j. A bargaining with equal strength between unions (together) and employers will require:

$$\sum_{i=1}^{n} \delta_i = 1 \tag{15}$$

F.O.C. will yield (12) and also [14]:

$$\sum_{i=1}^{n} U^i_W / U^i_L = \Pi_W / \Pi_L = - \sum_{i=1}^{n} L_i \; / [P\, F_L - W] \tag{16}$$

2.5. Final remarks

The comparison of the four forms for a union duopoly can be found in Martins & Coimbra (1997) [15]; without a particular form for the union utility function and labor demand not much can be add. Again, the main important features of the solutions are:

- the equality of the sum of the marginal rates of substitution between wage and labor in the coalitions cases, i.e., with efficient cooperation among the unions - as opposed to equality of each of such rates when unions compete as Cournot - to the slope of labor demand - or Π_W / Π_L -, thus suggesting a higher wage relative to employment when unions collude.

- a Stackelberg leader would pick a point where its marginal rate of substitution U^1_L / U^1_W is higher than the followers', suggesting a higher

[14] These properties would also arise from the solution of the problem

$$\underset{L_1, L_2, \dots, L_n, W}{\text{Max}} \quad U^1(L_1, W)$$

$$\text{s.t.:} \quad U^j(L_j, W) \geq \bar{U}^j \quad , \quad j = 2, 3, \dots, n$$

$$\Pi (\sum_{i=1}^{n} L_i, W) \geq \bar{P}$$

[15] See Table 1 of that work for a summary of the marginal rate of substitution conditions.

L_1 than if she were a follower - and higher than the followers' employment if the utility functions of all the unions are similar.

- efficient cooperation with the employer side leads to an equilibrium where wages are higher than the marginal product of labor - as it occurs in the traditional contract curve solution when there is only one union.

3. An analytical example

Assume that unions maximize the special case of the Stone-Geary utility function:

$$U^i(L_i,W) = W^{\theta_i} L_i^{(1-\theta_i)} \tag{17}$$

$\gamma_i = (1 - \theta_i) / \theta_i$ represents union i's relative (to wage) preference for employment. Whenever necessary, a linear demand schedule is going to be considered:

$$W = a - b \left(\sum_{i=1}^{n} L_i\right) \tag{18}$$

3.1. Oligopoly equilibrium

From F.O.C., we can derive:

$$[\theta_i / (1 - \theta_i)]\, s_i = \eta \tag{19}$$

where denotes union i´s employment share, i.e., $s_i = L_i / L$, and:

$$s_i = [(1 - \theta_i) / \theta_i] / \left[\sum_{j=1}^{n} (1 - \theta_j) / \theta_j\right] \tag{20}$$

$$\eta = 1 / \left[\sum_{i=1}^{n} (1 - \theta_i) / \theta_i\right] \tag{21}$$

Therefore:

Proposition 1: With Stone-Geary utility functions as above in a oligopoly union

1. the equilibrium employment share of each union is independent of the form of the production function (i.e., of labor demand)

2. the share of employment of each union will be equal to the weight of γ_i in total sum of γ_i's.

3. the market will lead to an equilibrium where the elasticity of demand is equal to the inverse of the total sum of the θ_i's.

The reaction functions are of the type:

$$L_i = a(1-\theta_i)/b - (1-\theta_i)(L-L_i) \tag{22}$$

Solving for L_i as a function of L:

$$L_i = (a/b)(1-\theta_i)/\theta_i - (1-\theta_i)L/\theta_i \tag{23}$$

Summing over i, we get:

$$L = \sum_{i=1}^{n} L_i = a/b \sum_{i=1}^{n} (1-\theta_i)/\theta_i - L \sum_{i=1}^{n} (1-\theta_i)/\theta_i \tag{24}$$

Solving for L, we get:

$$L = (a/b)\left[\sum_{i=1}^{n} (1-\theta_i)/\theta_i\right] / \left[1 + \sum_{i=1}^{n} (1-\theta_i)/\theta_i\right] \tag{25}$$

Using the labor demand schedule, we conclude that:

$$W = a / \left[1 + \sum_{i=1}^{n} (1-\theta_i)/\theta_i\right] \tag{26}$$

Replacing (25) in (23),

$$L_i = (a/b)\left\{[(1-\theta_i)/\theta_i] / \left[1 + \sum_{j=1}^{n} (1-\theta_j)/\theta_j\right]\right\} \tag{27}$$

We can therefore conclude that:

Proposition 2: In an oligopoly of unions with Stone-Geary preferences and linear labor demand:

1. $\gamma = \sum_{i=1}^{n} (1-\theta_i)/\theta_i$ is a measure of aggregate union preference for labor relative to wage.
2. Equilibrium aggregate employment is positively related to γ and negatively related to b, the slope of the demand.
3. The wage is inversely related to γ.

4. Each union's employment is positively related to $\gamma_i = (1 - \theta_i) / \theta_i$, the measure of the union preference for employment relative to wage.

Let us consider a symmetric equilibrium.

If all unions optimize the same utility function, then $\theta_i = \theta$, i=1,2,...,n. In this case, we will have that:

$$L = (a / b) \; 1 / \{ 1 + \theta / [n (1 - \theta)] \} \tag{28}$$

$$W = a / [1 + n (1 - \theta) / \theta] \tag{29}$$

$$L_i = (a / b) \{ 1 / [n + \theta / (1 - \theta)] \} \tag{30}$$

We therefore see that:

Proposition 3: In an oligopoly of unions with Stone-Geary preferences and linear labor demand:

1. W and L_i decrease with n; therefore, also W L_i, each union's wage bill will decrease with n.
2. L increases with n. But one can show that WL will decrease with n:
3. As $n \to \infty$, $W \to 0$ and $L \to a / b$ ($WL \to 0$).
4. n = 1 solves the monopoly union problem.

3.2. Stackelberg Equilibrium

We want to analyze the situation where a union, say 1, acts as leader and the others are followers. (6) and (8) hold. (6) yields:

$$L_i = a (1 - \theta_i) / b - (1 - \theta_i) (L_1 + \sum_{j=2}^{n} L_j - L_i), \quad i = 2,3,...n \tag{31}$$

Solving for L_i,

$$L_i = [a / b - (L_1 + \sum_{j=2}^{n} L_j)] (1 - \theta_i) / \theta_i, \quad i = 2,3,...n \tag{32}$$

Summing over i = 2,..,n

$$\sum_{i=2}^{n} L_i = [a / b - (L_1 + \sum_{j=2}^{n} L_j)] \sum_{i=2}^{n} (1 - \theta_i) / \theta_i \tag{33}$$

Solving in order to $\sum_{i=2}^{n} L_i = \sum_{j=2}^{n} L_j$,

$$\sum_{i=2}^{n} L_i = (a / b - L_1) \sum_{i=2}^{n} (1 - \theta_i) / \theta_i / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i] \qquad (34)$$

Replacing in (32) we get

$$L_i = [(1 - \theta_i) / \theta_i] (a / b - L_1) / [1 + \sum_{j=2}^{n} (1 - \theta_j) / \theta_j] , i = 2,3,...n \qquad (35)$$

Maximizing union 1's utility function with respect to the system yields:

$$L_1 \theta_1 b / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i] = (1 - \theta_1) W = \qquad (36)$$

$$= (1 - \theta_1)\{a - b L_1 - b \sum_{i=2}^{n} (1 - \theta_i) / \theta_i (a/b - L_1) / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i\}$$

Solving for L_1 :

$$L_1 = (1 - \theta_1) a/b \qquad (37)$$

$$W = a \theta_1 / \{[1+ \sum_{i=2}^{n} (1 - \theta_i) / \theta_i]\} \qquad (38)$$

For the followers:

$$L_i = (a/b) [(1 - \theta_i)/ \theta_i] \theta_1 / \{[1+ \sum_{j=2}^{n} (1 - \theta_j) / \theta_j]\} \qquad (39)$$

$$L = (a/b) \{ 1 - \theta_1 / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i]\} \qquad (40)$$

Comparing with (37)-(40) to (24)-(27) for given n, we conclude

Proposition 4: 1. W is now lower - L will be higher - than it was with no leader.

2. Each follower's quantity is lower than in the case of Cournot oligopoly.

Assume that $\theta_i = \theta$, i = 1,2,...,n. Then:

$$L_1 = (1 - \theta) a/b \qquad (41)$$

$$L_i = (a/b) (1 - \theta) / [1+ (n - 1) (1 - \theta) / \theta] \qquad (42)$$

$$L = (a/b) \{1 - \theta / [1 + (n-1)(1-\theta)/\theta]\} \quad (43)$$

and

$$W = a\,\theta / [1 + (n-1)(1-\theta)/\theta] \quad (44)$$

3.3. Efficient Bargaining

Assume, as usual, that $\bar{U}^i = 0$, i = 1, 2,...,n. In this setting, (12) yields:

$$\delta_i (1 - \theta_i) / L_i = \delta_j (1 - \theta_j) / L_j \quad (45)$$

Therefore:

$$s_i = \delta_i (1 - \theta_i) / \sum_{j=1}^{n} \delta_j (1 - \theta_j) \quad (46)$$

(13) yields:

$$\sum_{i=1}^{n} [\theta_i / (1 - \theta_i)]\, s_i = \eta \quad (47)$$

and:

$$\eta = \sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i (1 - \theta_i) = \quad (48)$$

$$= [\sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i] / \{[1 - [\sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i] =$$

$$= \bar{\theta} / (1 - \bar{\theta})$$

where

$$\bar{\theta} = \sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i \quad (49)$$

i.e., $\bar{\theta}$ is the weighted average of the θ_i's, the weights being the strength parameters in the coalition.

Proposition 5: With Stone-Geary utility functions and efficient bargaining between unions (i.e., coordination of union bargaining):

1. the equilibrium employment share of each union is independent of the form of the production function

2. the employment share of each union will be higher, the lower θ_i, the union's preference for wage, and the higher θ_i, the union's strength parameter.

3. the employment share of union i will be higher than in the Cournot game iff

$$\delta_i \theta_i > \sum_{j=1}^{n} \delta_j (1-\theta_j) \;/\; \sum_{j=1}^{n} (1-\theta_j) / \theta_j$$

4. the elasticity of demand will, in equilibrium, be higher than in the point where the Cournot game end.

5. The elasticity of aggregate demand will be equal to the monopoly union solution for a union the θ of which equals the weighted (by the δ_i's) average of the θ_i's.

If demand is linear, one can show that:

$$L_i = (a/b)\, \delta_i (1 - \theta_i) / \sum_{j=1}^{n} \delta_j \quad , \quad i = 1,2,..,n \tag{50}$$

$$L = (a/b) \sum_{i=1}^{n} [(1-\theta_i)\, \delta_i] / \sum_{i=1}^{n} \delta_i = (a/b)\, (1 - \bar{\theta}) \tag{51}$$

and

$$W = a \sum_{i=1}^{n} \theta_i \delta_i / \sum_{i=1}^{n} \delta_i = a\ \bar{\theta} \tag{52}$$

We can see that the aggregate labor market outcome is analogous to the monopoly union solution and does not depend on the number of unions in the industry (or in the economy) - suggesting that a higher number of unions will just have, at least on average, lower employment.

One can show that:

Proposition 6: In a equilibrium, with Stone-Geary union preferences and a linear demand schedule, with unions behaving cooperatively among themselves:

1. W is higher and aggregate employment is lower than in Cournot oligopoly.

2. Aggregate employment and wage is invariant to the number of unions in the economy, i.e., no matter how many unions there are in the economy, they will share the same aggregate employment.

3. In a symmetric equilibrium, i.e., $\theta_i = \overline{q} = \theta$ and $\delta_i = \delta$ for all i, W will be higher and employment will be lower than in the Stackelberg equilibrium.

4. In a symmetric equilibrium, each union's employment will be lower than in the Cournot case. Each follower will have a lower employment than in the Stackelberg case if $\theta > 0,5$.

3.4. Globally Efficient Bargaining

Then we can show that (45) and (46) hold and distribution of employment is the same as with only efficient bargaining among unions. Contract curve agreements will be such that:

$$W = (a - b\,L)\,\overline{\theta} / (2\,\overline{\theta} - 1) \tag{53}$$

Notice that this expression is equivalent to the contract curve with only one union. The number of unions does not affect (directly) the aggregate contract curve relation.

If $\overline{\theta} > 0,5$, for a positive (meaningful) wage, the marginal product of labor will be positive. If $\overline{\theta} < 0,5$ - in which case unions' preferences for employment (relative to wage) are very high -, for a positive wage, we must have in equilibrium aggregate employment till a point where the marginal product of labor is negative.

Therefore:

Proposition 7: In fully efficient bargaining with union Stone-Geary preferences and a linear demand curve, considering positive wage results, in equilibrium solutions:

1. If $\overline{\theta} > 0,5$, the marginal product of labor will be positive. If $\overline{\theta} < 0,5$, aggregate employment will be pushed till a point where the marginal product of labor is negative. This result is independent of unions relative strength to the employers side.

2. For given aggregate employment, contract curve agreements will imply a larger wage level the larger is the average union preferences for wage, $\overline{\theta}$.

3. The contract curve relation does not depend on the number of unions.

The effective solution depends on the δ_i's and turned out to be of difficult comparison with the previous cases. We concluded that, if unions maximize the wage bill, i.e., $\theta = 0,5$ and, for $\delta_i = \delta$ for all i:

$$L = (a / b) \tag{54}$$

$$L_i = (a / b)\,(1 / n) \tag{55}$$

The wage will (also) depend on the relative strength of the unions and employers.

4. Union entry

4.1. Introduction

Assume there is a fixed number of unions in the labor market. They set employment and membership is exogenous. If labor supply at the equilibrium wage is much higher than employment, unemployed workers may press wages down in order to get a job - that is, supply reacts. As we consider closed-shop agreements, and recalling some of the oligopoly symmetric solutions, aggregate employment increases with n, i.e., the number of unions. Then, it is reasonable to suppose that unemployed workers (unionized or not...) collude, form a (or other) new union(s) and push the wage down till they get employment.

We can derive the equilibrium number of unions, n^*, in several ways:

1. Reservation Wage Restriction.

We can assume there is a reservation wage W_r below which unions collapse. With entrance, wage decreases till this point is reached. Alternatively, W_r may represent a minimum wage floor, arising from (exogenous) general legislation.

2. Labor Supply Constraint.

Another form is to assume labor supply reacts directly, pressing down wages till unions are virtually irrelevant in wage and aggregate employment determination, only maintained through the legal closed shop requirements. There is a labor supply schedule (or a membership demand function), which will be reached.

Notice that this outcome guarantees that full-employment will be achieved.

3. Number of Unions Equals Demand.

Each employed worker behaves as a union, i.e., $L_i = 1$ (at least for non-leaders). Eventually, each union's utility function can be seen as representing each individual's preferences over wage W and probability of employment, L_i [16]; then, $n^* = L$ (for Cournot-Nash equilibrium; for Stackelberg, $n^* - 1 = L - L_1$) establishes the n^* that will guarantee full-employment (but only) of unionized workers.

In terms of insider-outsider theory, this corresponds to the solution of the maximum number of insiders the (closed) system supports - which depends on demand, and union preferences.

4. Individualistic Unions.

If each individual (not necessarily employed all the time...) behaves as an union and there are $\bar{L}$ individuals in the market, then a bound for n is $\bar{L}$.

[16] A similar interpretation of the household maximand can be found in Oswald (1979).

This could be seen as the limit solution, when the union utility function represents, as before in 3., the worker's preferences over the probability of (un)employment (and, indirectly, leisure) - wage mix . The final outcome would originate a Natural Rate of Unemployment, given by $(\bar{L} - \sum_{i=1}^{n} L_i) / \bar{L}$, as long as $L_i < 1$; this could be seen as underemployment solution, as $L_i > 1$ would correspond to overemployment cases.

This scenario is only meaningful in the Stackelberg case if we admit that it applies exclusively to the workers not employed in union 1, i.e., $n^* - 1 = \bar{L} - L_1$. It would be as if union 1 members would be full-employed insiders.

5. Minimum Employed Members Requirement.

A realistic scenario is that there is an exogenous - legally required - minimum number of employed members, M, that each union must exhibit to be constituted or considered representative for access to the closed-shop, i.e., $L_i \geq M$. Union entrance will occur till this bound is reached.

We can appreciate the dynamics of union formation in the Cournot environment. Or in the Stackelberg case. In the latter, at first glance, the scenario could be seen as describing the insider-outsider environment, with members of union 1 corresponding to insiders. However, insiders have the same wage as outsiders; only, the leader has now fixed employment - independent of n; but, as in the product market, the leader ends up by sustaining the reduction in quantity to support the wage.

Finally, note that, from (51)-(52) we conclude that with union collusion, the aggregate outcome is invariant to n. Nevertheless, conclusions can be drawn for some of the five cases.

To derive the equilibrium n, n*, we admit symmetric unions, i.e., $\theta_i = \theta$ and $\delta_i = \delta$ for all i. We ignore problems (indivisibilities) arising from treating n as a rational number. We also derive the final labor market outcome - W, L and L_i - for the endogenous n*.

4.2. Nash-Cournot equilibrium

The derivation of results starts from equations (28)-(30).

A. Reservation Wage Restriction.

Then, if all unions are similar, using (29), we will have union "entrance" till:

$$W = W_r = a / [1 + n(1 - \theta)/\theta] \tag{56}$$

That is, the maximum number of unions the market will support will be:

$$n^* = (a / W_r - 1)\ \theta / (1 - \theta) \tag{57}$$

n^* is positively related to the unions relative preference for wage and negatively related to the reservation or minimum wage. Replacing (57) in (28) and (30):

$$L = (a - W_r) / b \quad (58)$$

$$L_i = W_r \ [(1 - \theta) / \theta] / b \quad (59)$$

B. Labor Supply Constraint

We will have (28) and (29) and, say, a linear labor supply schedule (or a membership demand function):

$$W = c + d\,L \quad (60)$$

Solving for n^*:

$$n^* = [b\,(a - c) / (bc + da)] \ \theta / (1 - \theta) \quad (61)$$

The equilibrium wage and employment in this case will be the same as with no unions - i.e., supply equals demand.

$$L = (a - c) / (b + d) \quad (62)$$

$$W = (bc + da) / (b + d) = b \ (L/n^*) \ \theta / (1 - \theta) \quad (63)$$

$$L_i = [(1 - \theta) / \theta] \ (bc + da) / [b\,(b + d)] \quad (64)$$

We expect that $n^* < L$, i.e., each union will employ more than one individual, $L_i > 1$.

C. Number of Unions Equals Demand

If each employed worker behaves as the union, then, $n^* = L$ and

$$L_i = 1 \quad (65)$$

Using in (30), we can solve for:

$$n^* = L^* = a/b - \theta / (1 - \theta) \quad (66)$$

Notice that now n^* is negatively related with the unions relative preference for wage and negatively related to the inverse of the labor demand slope, b. Eventually, each union's utility function can be seen as representing each individual's preferences over wage W and probability of employment, L_i; then, condition (66) establishes the n^* that will guarantee full-employment of unionized workers.

Also

$$W = b \ \theta / (1 - \theta) \quad (67)$$

W will be the same as in (63) for n* = L.

D. Individualistic Unions

If each individual (not necessarily employed all the time...) behaves as an union and there are $\bar{L}$ individuals in the market, then a bound for n is $\bar{L}$ and (28)-(30) with n replaced by $\bar{L}$ will be the bounds for W, L and L_i - with L_i representing the equilibrium probability of employment of each individual. We will therefore have:

$$n^* = \bar{L} \tag{68}$$

$$L = (a / b) \; 1 / \{ 1 + \theta / [\bar{L} (1 - \theta)] \} \tag{69}$$

$$W = a / [1 + \bar{L} (1 - \theta) / \theta] \tag{70}$$

$$L_i = (a / b) \{ 1 / [\bar{L} + \theta / (1 - \theta)] \} \tag{71}$$

$L_i \leq 1$, iff

$$\bar{L} \geq (a/b) - \theta / (1 - \theta) \tag{72}$$

E. Minimum Union Employed Members Requirement

There is an exogenous minimum number of employed members, M, that each union must exhibit to be constituted. Then, entrance will occur till the point where:

$$L_i = M \tag{73}$$

Using (30) we conclude that

$$n^* = (a/b) (1/M) - \theta / (1 - \theta) = [a (1 - \theta) - b M \theta] / [b M (1 - \theta)] \tag{74}$$

It varies negatively with M and ◎ , the union preference for wage relative to employment.

$$L = (a/b) - M \theta / (1 - \theta) = [a (1 - \theta) - b M \theta] / [b (1 - \theta)] \tag{75}$$

and

$$W = b M \theta / (1 - \theta) \tag{76}$$

Proposition 8: 1. The equilibrium number of unions increases with the unions' relative preference for wage in the reservation wage and the labor supply constraint cases. It decreases when number of unions equals demand and when there is employed membership requirement.

2. A reservation wage level higher than the final equilibrium wage in all other cases will imply a smaller number of unions in equilibrium.

3. In general, we expect individualistic unions to originate a higher number of unions than any other case (provided only that $L_i > 1$ for the labor supply constraint).

4. We expect that the minimum employed members requirement implies the lowest equilibrium number of unions (provided only that the requirement is higher than the equilibrium employment per union of the other cases).

One can give an intuition for the result 1. of Proposition 7. In the first two cases, we have, in fact, a labor supply constraint: the reservation wage hypothesis corresponds to a infinitely elastic labor supply. Therefore, supply and demand determine total employment - and wage. In these circumstances, the number of unions can be seen as determined from wage equation (29), showing that the wage varies negatively with n and positively with θ - as expected; therefore the positive relation between n and θ follows immediately.

In the case where the number of unions is demand determined, the stronger are the insiders preferences for employment - weaker for wage - the larger will be aggregate employment - for given n. As in this case n is simultaneously determined by demand, we arrive at the conclusion that n* should increase with (1 - θ) - increase with θ.

The explanation for the minimum employed members requirement case can invoke the relation (30): in a Cournot equilibrium, each union's employment varies inversely with n and θ. Then, if L_i is fixed, the relation between n and θ is straightforward.

4.3. Stackelberg Equilibrium

To derive the equilibrium number of unions, we consider equations (41)-(44). Employment of the leader, L_1, does not depend on n and is always fixed and equal to:

$$L_1 = (1 - \theta)\, a/b \tag{77}$$

Considering:

A. Reservation Wage Restriction

Then:

$$n^* = \theta / (1 - \theta)\, [\, a\, \theta / W_r - (2\, \theta - 1) / \theta] \tag{78}$$

We can show that the new n^* will be smaller than for Cournot oligopoly iff $W_r / a < \theta$; from (44) we can see that this always occurs. Total employment is the same as in the Cournot case and given by (58).

B. Labor Supply Constraint

As we have seen, total employment and wage is fixed outside the control of the union - given by

$$L = (a - c) / (b + d) \tag{79}$$

$$W = (bc + da) / (b + d) \tag{80}$$

Then,

$$n^* = \theta / (1 - \theta) [a \theta (b+d) / (bc + ad) - (2 \theta - 1) / \theta] \tag{81}$$

C. Number of Unions Equals Demand

I.e., for outside unions $L_i = 1$.

$$n^* = a \theta / b - (2 \theta - 1) / (1 - \theta) \tag{82}$$

If $a/b > 1 / (1 - \theta)$, i.e., for the leader, $L_1 > 1$, which we implicitly assume, then the new n^* will be smaller than in the Cournot case.

In this case:

$$L = (1 - \theta) a/b + (n^* - 1) = a/b - \theta / (1 - \theta) \tag{83}$$

Therefore, total employment is maintained relative to Cournot oligopoly. Only a redistribution favoring the leader will take place.

D. Individualistic Unions

We admit the restriction applies to workers not employed in union 1 - the insiders which would have full-employment. We will therefore have:

$$n^* - 1 = \bar{L} - L_1 = \bar{L} - (1 - \theta) a/b \tag{84}$$

Then:

$$n^* = \bar{L} - L_1 + 1 = \bar{L} + 1 - (1 - \theta) a/b \tag{85}$$

n^* will be smaller than in the Nash-Cournot case. Also:

$$L = (a / b) (1 - \theta / \{1 + [\bar{L} - (1 - \theta) a/b] (1 - \theta) / \theta]\}) \tag{86}$$

$$W = a\,\theta \;/\; \{1 + [\bar{L} - (1 - \theta)\, a/b]\, (1 - \theta) / \theta]\} \tag{87}$$

$$L_i = (a / b)\, \theta \,/\, \{\theta / (1 - \theta) + [\bar{L} - (1 - \theta)\, a/b]\} \tag{88}$$

As in the Nash-Cournot case, $L_i \leq 1$ iff

$$\bar{L} \geq a/b - \theta / (1 - \theta) \tag{89}$$

E. Minimum Union Employed Members Requirement

M is going to restrict the n-1 followers:

$$L_i = M \quad , \quad i = 2, 3, ..., n \tag{90}$$

We can see then that:

$$n^* = \theta\, a/(b\, M) - (2\,\theta - 1) / (1 - \theta) \tag{91}$$

From (43),

$$L = (a/b) - M\,\theta / (1 - \theta) \tag{92}$$

From (44),

$$W = b\, M\, \theta / (1 - \theta) \tag{93}$$

As in previous cases, n^* is smaller than in the analogous Cournot solution iff $\theta > W/a = (b\, M /a)\, \theta / (1 - \theta)$, which - from (44) - must always occur. However, the aggregate solution (L, W) is the same as in the corresponding Cournot case.

Proposition 9: 1. Stackelberg equilibrium:

1. in general, will originate a smaller number of unions than the corresponding Nash-Cournot case.
2. will lead to the same aggregate employment and equilibrium wage than the corresponding Nash-Cournot case.

4.4. Efficient Bargaining

We consider equations (50)-(52) and assume the symmetric case where $\theta_i = \theta$ and $\theta_i = \theta$ for all i. It will always be the case that:

$$L_i = (a/b)\;(1 - \theta) / n = L / n \quad , \quad i = 1,2,...,n \tag{94}$$

$$L = (a/b)\,(1 - \theta) \tag{95}$$

and

$$W = a\,\theta \tag{96}$$

Therefore, total employment and wage are fixed and do not depend on n.

B. Labor Supply Constraint

In this case, denote the supply wage, W^S and effective supply L^S at the efficient bargaining result, which is independent of n:

$$W^S = c + d\,(a/b)\,(1 - \theta) \tag{97}$$

$$L^S = (a\,\theta - c)\,/\,d \tag{98}$$

If $W = a\,\theta > c + d\,(a/b)\,(1 - \theta)$, there will be involuntary unemployment at rate:

$$(L^S - L)\,/\,L^S = \{a\,[b\,\theta - d\,(1 - \theta)] - c\,b\}\,/\,[b\,(a\,\theta - c)] \tag{99}$$

However, if $W = a\,\theta < c + d\,(a/b)\,(1 - \theta)$, there won't be enough supply.

Alternatively, we can admit that in any case number of people

$$n^* = L^S = (a\,\theta - c)\,/\,d \tag{100}$$

and number of available jobs are given by (95). Each "supply unit" will work:

$$L\,/\,L^S = (a/b)\,(1 - \theta)\,/\,[(a\,\theta - c)\,/\,d] \tag{101}$$

This may be smaller than 1 (corresponding to the involuntary unemployment case), or larger than 1.

Notice that this case is not directly comparable to the Cournot or Stackelberg cases, once there is now unemployment.

C. Number of Unions Equals Demand

If each employed worker behaves as the union, then, $n^* = L$ and

$$L_i = 1 \tag{102}$$

Then:

$$n^* = (a/b)\,(1 - \theta) \tag{103}$$

D. Individualistic Unions

We will therefore have, as long as $L_i \leq 1$:

$$n^* = \bar{L} \tag{104}$$

and

$$L_i = (a/b)\ (1 - \theta) / \bar{L}\ , \qquad i = 1,2,\ldots,n \tag{105}$$

$L_i \leq 1$, iff:

$$(a/b)\ (1 - \theta) \leq \bar{L} \tag{106}$$

E. Minimum Union Employed Members Requirement

Then:

$$L_i = M \tag{107}$$

Using (94) we conclude that

$$n^* = (a/b)\ (1 - \theta) / M \tag{108}$$

Proposition 10: Efficient bargaining:

1. will imply that the equilibrium number of unions will vary with unions preference for wage as in the Cournot case, except in the minimum membership requirement case (in which they now vary inversely).
2. will originate a smaller aggregate employment and higher wage than any Nash-Cournot or Stackelberg cases (except for the labor supply constraint).
3. will imply a smaller number of unions than the corresponding Nash-Cournot case in the minimum employed members requirement iff $\theta < 0{,}5$ (a larger number iff $\theta > 0{,}5$); a smaller number in number of unions equal demand.

5. Summary and conclusions

This paper gathers some notes and enlargements to the standard collective bargaining problem in which unions maximize utility; we oriented the analysis to model union formation.

The research started by confronting different scenarios of union and firm(s) strategic behavior. The results - some are summarized in Table 1 - are an extension of the duopoly case, previously studied. A specific example using Stone-Geary union utility functions is derived - see Table 2 for some of the main labor market outcomes. We conclude that efficient cooperation among unions can be seen as a union composite behaving as a monopoly union; cooperation between unions and employers reproduce contract curve agreements for the one union solution.

The symmetric equilibrium (with an exogenous number of unions) allows us now to infer that with non-cooperative (among themselves) union behavior - as expected -, the equilibrium wage will decrease with the number of unions and employment will move in the opposite direction. Yet, with efficient bargaining among unions, aggregate outcomes are invariant to the number of unions in the industry.

The equilibrium (endogenous) number of unions - which will presumably rise while there is unemployment - is studied for five cases: there is a reservation or minimum wage restriction; a standard labor supply constraint closes the model; number of unions equals demand; individualistic unions, the number of which is exogenous and equal to the number of individuals in the economy; there is a legal minimum employed members requirement. Minimum employed membership requirements, with free union entry, partly restore union effectiveness in wage lifting. The results are summarized in Table 3. Interestingly, it is not always the case - as one would expect - that number of unions increases with the unions' preferences for wage relative to employment. Number of unions will be larger (or at least equal) in Cournot-Nash than in Stackelberg environments; both will have the same aggregate outcome, i.e., wage and total employment.

Efficient bargaining will support, in general, a smaller (or not larger) number of unions than Cournot, except when minimum employed membership requirements are imposed (With a labor supply constraint, results are not comparable). Aggregate employment will be smaller in efficient bargaining - and wage higher - than in Cournot or Stackelberg cases in the final equilibrium with endogenous number of firms.

Table 1. *Marginal Rate of Substitution Conditions*

	Equation	Efficiency Locus
A. Cournot-Nash	(5)	$U^i_L / U^i_W = - W_L$, $i = 1,2, \ldots, n$
B. Stackelberg	(9)	$U^1_L / U^1_W = (1 + \Sigma_{j=2}^{n} dR^j / dL_1) U^i_L / U^i_W =$ $= - (1 + \Sigma_{j=2}^{n} dR^j / dL_1) W_L$, $i = 2, 3, \ldots, n$
C. Efficient Union Cooperation	(13)	$\Sigma_{i=1}^{n} U^i_W / U^i_L = - L_W$
D. Globally Efficient Cooperation	(16)	$\Sigma_{i=1}^{n} U^i_W / U^i_L = \Pi_W / \Pi_L =$ $= - (\Sigma_{i=1}^{n} L_i) / [P F_L - W]$

Table 2. *Equilibrium Outcomes for Stone-Geary Union Utility Functions*

Variable	Cournot Oligopoly	Stackelberg Oligopoly
L_1	(27) $(a/b)\,(1/\gamma_1)\,/\,[1+\sum_{j=1}^{n}(1/\gamma_j)]$	(37) $a\,(1-\theta_1)\,/\,b$
L_i	(27) $(a/b)\,(1/\gamma_i)\,/\,[1+\sum_{j=1}^{n}(1/\gamma_j)]$	(39) $(a/b)\,[(1-\theta_i)/\theta_i\,]\,\theta_1\,/\,\{[1+\sum_{i=2}^{n}(1-\theta_i)\,/\theta_i\,]\}$
L	(25) $(a\,/b)\,[\sum_{i=1}^{n}(1-\theta_i)\,/\,\theta_i\,]\,/\,[\,1+\sum_{i=1}^{n}(1-\theta_i)\,/\theta_i]$	(40) $(a/b)\,\{1-\theta_1/[1+\sum_{i=2}^{n}(1-\theta_i)\,/\theta_i\,]\}$
W	(26) $a\,/\,[\,1+\sum_{i=1}^{n}(1-\theta_i)\,/\,\theta_i\,]$	(38) $a\,\theta_1\,/\{[1+\sum_{i=2}^{n}(1-\theta_i)\,/\theta_i]\}$

Variable	Efficient Union Cooperation	Globally Efficient Cooperation (for $\theta_i = 0.5$; $\delta_i = \delta$)
L_i	(50) $(a/b)\,\delta_i\,(1-\theta_i)\,/\sum_{j=1}^{n}\delta_j$	(55) $(1/n)\;a\,/\,b$
L	(51) $(a/b)\,\sum_{i=1}^{n}[(1-\theta_i)\,\delta_i]\,/\,\sum_{i=1}^{n}\delta_i \;=\; (a/b)\,(1-\bar{\theta})$	(54) $a\,/\,b$
W	(52) $a\,\sum_{i=1}^{n}\theta_i\delta_i\,/\sum_{i=1}^{n}\delta_i = a\,\bar{\theta}$	(Dependent on δ)

Table 3. *Equilibrium Number of Unions*

	Cournot-Nash	Stackelberg	Efficient Bargaining
A. Reservation Wage Restriction	(57) $(a/W_r - 1)\ \theta/(1-\theta)$	(78) $\theta/(1-\theta)\ [\ a\theta/W_r -$ $- (2\ \theta - 1) / \theta]$	-
B. Labor Supply Constraint	(61) $[b\ (a - c) /$ $/ (bc + da)]\ \ \theta / (1 - \theta)$	(81) $\theta / (1 - \theta)\ [a\ \theta$ $(b+d) / (bc + ad)\ -$ $- (2\ \theta - 1) / \theta]$	(100) $(a\ \theta - c) / d$
C. Number of Unions Equals Demand	(66) $a/b\ - \theta / (1 - \theta)$	(82) $a\ \theta / b -$ $- (2\ \theta - 1) / (1 - \theta)$	(103) $(a/b)\ (1 - \theta)$
D. Individualistic Unions	(68) $\bar{L}$	(85) $\bar{L} - L_1 + 1 =$ $= \bar{L} + 1 - (1 - \theta)\ a/b$	(104) $\bar{L}$
D. Minimum Union Member Requirement	(74) $a/(bM) - \theta/(1-\theta)$	(91) $\theta\ a/(b\ M) -$ $- (2\ \theta - 1) / (1 - \theta)$	(108) $(a/b)\ (1 - \theta)\ M$

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