Clower’s Dual-Decision Hypothesis is economics

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Abstract. Though Wu (2017) has shown Clower’s Dual Decision Hypothesis leading to Keynes’ change in saving (and disequilibrium) conclusion, it is important to compare Clower’s budget constraint approach with other models, including those found in Hall’s consumption theorem and similar approach. In Clower, by assuming that, consumers may not satisfy the budget constraint, one cannot automatically assume Hall’s consumption theorem to hold. And, by showing how households need to optimize contingent on the satisfaction of their budget constraint, Clower was, in effect, creating a feedback mechanism.

Keywords. Keynes, Clower, Keynesian, Disequilibrium, Dual Decision Hypothesis, Consumption, Martingale, Saving, Growth, Income, Trade, Feedback.

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1. Introduction

Wu (2017) has shown that Clower’s Dual Decision Hypothesis (DDH) provided the essential dual-optimization technique to arrive at Keynes’ change in saving result, which effectively unraveled the consumption theory of the General Theory and consequently Keynes’ disequilibrium thesis. Still, there are other elements in DDH, specifically the budget constraint, which may be interesting to review. Assuming consumption under certainty for an individual who lives for T periods and whose lifetime utility is

\[ U = \sum_{t=1}^{T} u(C_t), u'(\bullet) > 0, \quad u'''(\bullet) < 0 \] (1)

Where \( u(\bullet) \) is an instantaneous utility function and \( C_t \) is consumption in period \( t \). The individual has initial wealth of \( A_0 \) and labor income \( Y \). To solve the optimization most economists assume some form of constant budget constraint, e.g., Hall (1978) assumed that, (for zero interest rate) individual’s lifetime budget constraint is

\[ \sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t \] (2)

But what if as Clower’s Dual Decision Hypothesis (DDH) argued the individual realized current income (\( Y' \)) were less than the notional (planned) income (\( Y \)),

\[ \sum_{t=1}^{T} Y_t' > \sum_{t=1}^{T} Y_t \] (3)

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That is, for short and long periods, the real budget constraint actually was

$$\sum_{t=1}^{T} C_t > A_0 + \sum_{t=1}^{T} Y'_t$$

(4)

Although DDH was from early 1960s, we still have many unanswered questions:

1. Does consumption optimization occur if the budget constraint is violated?
2. Does Hall’s consumption theorem hold if budget constraint is violated?
3. What is the simplest optimization mechanism?
4. What is the budget constraint mechanism under DDH?

After all these years, what is the contribution of Clower’s DDH to economics?

2. The budget constraint

Most consumption models state that a consumer maximizes

$$\sum_{t=0}^{\infty} b^t [u_0 + u_1 c_t + \frac{u_2}{2} c_t^2], \quad 0 < b < 1, \quad u_0, u_1, u_2 > 0$$

(5)

subject to

$$A_{t+1} = R [A_t + y_t - c_t]$$

(6)

and where $y_t$, under a stochastic process, is $E_y$.

Where, $c$ is consumption, $A$ is non-human assets, $y$ is labor income, $R$ is gross rate of return (all at the beginning of period), $E$ is expectation, $t$ is time.

Under the Euler approach, the optimal consumption is

$$c_{ru} = (1 - R') [A_{ru} + \sum_{j=n}^{\infty} \left( \frac{1}{R} \right)^j E_{t+n} y_{t+j}]$$

(7)

Eq. (7) provides theoretical support to Permanent Income Hypothesis (PIH), i.e., decisions are made based on present and estimate of future incomes or lifetime income.

The critical issue is the budget constraint and here are two examples of it:

2.1. Hall’s budget constraint

In Hall’s interpretation of life-cycle/permanent income hypothesis, he assumed that the (lifetime) budget constraint is given by

$$A_t = \sum_{t=1}^{T} C_t + \sum_{t=1}^{T} Y_t$$

(8)

In certainty consumption, the Lagrangian is

$$\mathcal{L} = \sum_{t=1}^{T} u(C_t) - \lambda \left( A_0 + \sum_{t=1}^{T} C_t + \sum_{t=1}^{T} Y_t \right)$$

(9)
The first order condition for \( C_t \) is

\[
\frac{\partial \mathcal{L}}{\partial C_t} = 0 : \ u^t C_t - \lambda = 0 \quad (10)
\]

As neither the utility function nor the budget constraint changes over time, the first order condition for any subsequent \( C_{t+1} \) is

\[
\frac{\partial \mathcal{L}}{\partial C_{t+1}} = 0 : \ u^t C_{t+1} - \lambda = 0 \quad (11)
\]

Under these exacting assumptions, the Lagrange multiplier \( \lambda \) is always the same and

\[
u^t C_{t+1} = u^t C_t \quad (12)\]

Similarly, applying the life-cycle/Permanent Income consumption under uncertainty, Hall has shown that

\[
E_t u^t C_{t+1} = f(u^t C_t) \quad (13)
\]

The critical assumption in Hall is lifetime budget constraint, “note that the new consumption strategy also satisfies the budget constraint (p. 986).” Can this type of budget constraint truly relate to PIH? Arguably, lifetime income in eq. (7) is not equal to lifetime budget constraint.

2.2. Fisher’s two-period budget constraint

Budget constraint for period 1 is

\[
b_1 + y_1 = c_1 + a_1 \quad (14)
\]

where \( b \) is beginning bank balances and \( a \) is asset,

for period 2 accumulated assets are multiplied by a gross interest rate \( R = 1 + r \)

\[
b_2 = a_1 R \quad (15)
\]

Thus the intertemporal budget constraint (IBC) is

\[
c_1 + c_2 / R \leq y_1 + y_2 / R + b_1 \quad (16)
\]

Assuming IBC, it is also possible to arrive at Hall’s consumption theorem.

Again, budget constraint is satisfied over time by the introduction of \( R \). IBC and lifetime budget constraint are just assumptions.

3. Clower’s Dual-Decision Hypothesis

Clower questioned whether “buying and selling are all carried out simultaneously… planned sales and purchases cannot possibly be true of realized sales and purchases, unless the system as a whole is always in a state of equilibrium.” Further, “differences between realized and planned purchases and sales of individual households may properly be supposed to occur more or less at random.”

Clower stated that, initially, households will maximize the preference function

\[
U(d_1, \ldots, dm; s_{m+1}, \ldots, s_t) \text{ subject to the budget constraint}
\]

\[ \sum_{i}^{m} p_i d_i - \sum_{j}^{n} p_j s_j - r = 0 \]  
(17)

where \( r \) is a profit variable.

If realized income is less than the notional income,

\[ \sum_{j}^{n} p_j s_j < \sum_{j}^{n} p_j s_j \]  
(18)

Clower stated that a “‘second round of decision making is indicated: namely, maximize’”

\[ U(d_1, \ldots, d_m; s_1, \ldots, s_n) \] subject to the modified budget constraint (based on new information available),

\[ \sum_{i}^{m} p_i d_i - \sum_{j}^{n} p_j s_j - r = 0 \]  
(19)

Then, for Clower, the new budget constraint is

\[ \sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t' \]  
(20)

where \( Y_t' < Y \) for period \( t \) or any period, \( t_1, \ldots, t_t \).

To maximize \( U(c, y) \), subject to \( g(c, y) = 0 \), the Lagrange function is

\[ \mathcal{L}(c, y; \lambda) = U(c, y) - \lambda g(c, y) \]  
(21)

Even if one were to assume that \( U(c, y) \) is only a function of \( c \), the first order condition for \( C_t \) is now

\[ \frac{\partial \mathcal{L}}{\partial C_t} = 0 : u'C_t - \lambda' = 0 \]  
(22)

and the first order condition for \( C_{t+1} \) is

\[ \frac{\partial \mathcal{L}}{\partial C_{t+1}} = 0 : u'C_{t+1} - \lambda'' = 0 \]  
(23)

Given that under DDH, the first budget constraint may not be satisfied, if \( \lambda' \neq \lambda'' \) then

\[ u'C_{t+1} \neq u'C_t \]  
(24)

and

\[ E_f u'C_{t+1} \neq f(u'C_t) \]  
(25)

The issue of budget constraint is not just one of theoretical matter; it has also an important implication in system dynamics.

4. Clower’s DDH and feedback

In recent years, Clower’s DDH relevance has declined and few economists have associated DDH to feedback or consumption feedback. For instance, while Chiarella et al., (2012), in a recent two volume perspectives of Keynesian
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macroeconomics, which provide many examples of feedback in macroeconomics, dual decision hypothesis is quickly dismissed in a single paragraph, “one need not to be convinced by microeconomic of the dual decision hypothesis… they can always realize the optimal consumption/labor supply decision they derive from given wages, prices and intertemporal choice given by the Euler equation, in complete isolation from all other economic information and macroeconomic feedback effects affecting next period’s economic outcome, the hypothesis that we have sketched above isto be preferred. The great macroeconomists in the past knew this and it in fact was already part of the pre-Keynesian Neoclassical analysis of Pigou and others when they studied the causes of unemployment in the macroeconomy (p. xxxviii).” Similarly, one can make the argument that, since Hall’s consumption theorem, DDH and feedback in consumption optimization has been effectively marginalized.

For Clower, the primary concern in any optimization is to question whether the budget constraint in eq. (17) is satisfied or not. In case the first optimization is not satisfied, a feedback is required so that realized income inputs are applied in a new optimization. Only then one is allowed to proceed to the next step. This dual-step process can be best described by the following flowchart:

**Figure:** Optimization with Budget constraint with and without Clower’s DDH Feedback

In contrast, under the unconvincing assumptions resulting from life-cycle-permanent income hypothesis, budget constraint models, such as lifetime budget constraint and the IBC, have systematically failed to take into account temporary and permanent violations. Arguably, the notion that a systems hould have no feedback is as alien as modern technology; just imagine a software or an electronic circuit that negates the execution of a feedback. As Leijonhufvud (1968) wisely concluded, “when the economic system fails to behave in the manner of the Classical model, it is not due simply to the absence of the feedback mechanisms assumed by the Classics.”

One could almost split Clower’s Keynesian counter-revolution article into two parts: one about Say’s principle and Walras’ law and DDH. But, what makes Clower’s DDH essential to economics is threefold. (1) Clower’s DDH provided the essential technique to reach Keynes’ change in saving result (Wu, 2017), and has finally unraveled the consumption theory of the General Theory and consequently Keynes’ disequilibrium; Keynes/DDH has also satisfied the recent prerequisite of micro foundations for macroeconomics. (2) By offering an alternative model,
Clower’s DDH was challenging the established notion that budget constraint should always be satisfied, a questionable practice found in Hall’s consumption theorem. (3) Regardless of whether anyone ever achieves its maximization and as the feedback above shows, the intrinsic mechanism of DDH with feedback is simple enough for the average household to intuitively understand and utilize on daily basis. For these reasons, the generalized Clower’s famous quote should be: “households either had a DDH in the back of their minds, or most of the consumption models are theoretical nonsense.”

References

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