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Economies of depth: Technology, market structure and vertical integration

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Abstract. In this paper, a formal model of technology-based vertical market structure is interacted with different environments of downstream and upstream strategic behavior. The research is innovative on four grounds: on the one hand, it explicitly establishes mathematical conditions for the assessment of vertical arrangements at the firm level, rigorously defining – as a peculiar production (in)externality - economies of depth. Secondly, it provides equilibrium and welfare comparisons of vertically integrated and non-integrated or decentralized market structures under different assumptions on the intervening productive agents competitive behaviour – final output producers, intermediate product suppliers, and primary factor owners. The exercises highlight transmission mechanisms of market power through intermediate product industries. The consequence of each particular market failure is studied independently (or in absence) of the others, considering ceteris paribus deviations of conduct relative to the competitive paradigm of each side of a transaction. Thirdly, the analysis shares a common well-defined representation of the underlying technologies, relying on duality and other production theory properties with respect to factor prices and usage – allowing for substitutability as for complementarity. Finally, the role of pre-commitment or contractual arrangements – vertical restraint clauses - is discussed in connection to the sequencing or hierarchy of the decision process. A final qualification of the effect uncertainty in upstream markets on equilibrium outcomes is suggested.
Keywords. Economies of depth; Internalities; Vertical integration; Vertical mergers; Intermediation; Intra-industry trade. Economic (mechanism) design: Short and long-run reaction functions; Pre-commitment; Vertical restraints; Trade unions. Monopsony in primary factor markets.
JEL. L14; L22; L42; D23; D62; D40; J50; J42.

1. Introduction

Theory and measurement of economies of scale are well documented in economics literature. Even economies of scope were subject to rigorous mathematical study under multiproduct technology theory (Chambers, 1988). Firms' vertical arrangement does not seem to have benefited from identical analysis: going over the vertical structure literature, a common feature is the usual neglect of the reference to a well-defined upstream (sellers of intermediate products) – or downstream (buyers of intermediate products and sellers of final products) – production

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function, connected to the use of outside primary factors; the analysis usually stops at cost functions. Of course, vertical integration was modelled and connected to merger occurrence – in a context of a variety of plausible reasons. Yet, a link to final input and factors "derived" demand is generally absent. Which may be irrelevant for most analysis – and we will recover some well-known results of previous studies. But by proceeding to it, we get a wider understanding of transmission of market power over an industry, study the less popular case of complementarity, provide simpler – or novel - rationales for particular arrangements such as vertical restraints, and proceed to a different type of assessment of vertical mergers. In particular, it allows the study of uncompetitive primary factor markets – a subject usually not associated to vertical integration.

The first thing that is acknowledged in this research is the potential indeterminacy of vertical market arrangements under conditions of competitive markets and absence of (positive or negative) technological "economies of depth" – nesting or "chain economies" -, these referred to the use and internal production capacity of primary inputs by downstream technologies. Yet, even if those economies – adding up Perry's (1989) technological and transactional economies (Williamson, 1975) – exist on a (upstream)firm-to-(downstream)firm basis, the vertically decentralized market may still be able to provide the inputs at a lower cost – if the optimal scale of the downstream firm requires a very different input quantity than the competitive scale of upstream producers -, justifying the literature on vertical integration (Tirole 1988; Perry, 1989; Hay & Morris 1991; Martin, 1993; and Guerra, 1997 for surveys) relying on a constant returns to scale (CRS) assumption for the upstream technology or on the existence of small number of intermediate input manufacturers.

A second type of determinants of the vertical arrangement of the industry is external to the downstream technology and rather relies on the market structure of the surrounding environment – the number and strategic behavior of the intermediate input manufacturers and the substitutability between the intermediate inputs in the downstream technology (Vernon & Graham, 1971; Schmalensee, 1973; and Westfield, 1981 assess upstream monopoly); of sellers of the same final output (Perry, 1978) studies monopsony towards upstream producers); market power of the owners of primary factors used in the industry.

In a decentralized – or not… - industry, a new market layer is introduced, and the sequencing of the multiple decision processes may exhibit different features. Apparently, perfect competition allows alternative hierarchies to generate equivalent outcomes; that is not the case with deviations from the competitive paradigm. Vertical restraints are then a source of such conditioning; this may be based on contractual arrangements, but also be physically or technologically induced. They are sometimes suggested by the existence of 'externalities' to the upstream maximization problem (Mathewson & Winter, 1984). Also Irmen (1998) for a survey of related developments); by uncertainty or asymmetric information (Rey & Tirole's 1986 argument; see Katz, 1989 for a survey); and, of course, also important for the comparison between decentralized and vertically integrated prototypes. Usually, they are considered to be imposed by a uncompetitive manufacturer on a retailer; we note that the implicit quantity (under quantity-fixing, QF) or price limiting level (under resale price maintenance, RPM [2]) may, nevertheless, ultimately be set by the downstream entity and study their effect in this light: their reaction-limiting role, potentially advantageous against uncompetitive opponents. We expand the argument to interpret tying arrangements in contexts of multiproduct bargaining.

A final application is directed towards including uncertainty in the framework. We study upstream market uncertainty (Carlton 1979 analyses final output demand uncertainty), extending Arrow's (1975) appraisal, and consider the simpler example of exogenous fluctuations affecting decisions in four typical contexts (Aiginger's, 1987 typology, also used in Martins, 2004): quantity-quality, with or without ex-post flexibility towards quantity decisions and price formation.

The research on the effects of uncompetitive conduct is illustrated with duopolistic examples and directed towards the recognition of familiar reaction function (geometric) analysis of the resulting (simultaneous…) Nash equilibria.

Notation with respect to the intervening firms' tecnologies and technical economies of depth are defined in section I. Section II inspects properties of reaction function equilibrium of uncompetitive upstream producers, with and without vertical restraints, and section III those of intermediate product buyers. In section IV, similar analysis is conducted for an industry facing uncompetitive primary factor pricing; monopsonistic behavior in such markets is then dealt with in section V. In section VI, uncertainty in upstream markets qualifies the likelihood of emergence of in-house production. The exposition ends with a brief summary in section VII.

2. Technology

Let there be a final product y in the economy which can be produced through the function:

$$y = f(x^y{}_1, x^y{}_2, L_y) \tag{1}$$

L_y denotes quantity of a primary factor of production, $x^y{}_1$ and $x^y{}_2$, of two intermediate products. These are produced by independent firms according to specific product functions requiring production factors L_i:

[2] Gilligan (1986) summarizes its role among others as a cartel discipline device. Under downstream monopoly, we will highlight its ability as a competitive promotion one.

$$x_i = g^i(L_i) \quad , \quad i = 1, 2 \tag{2}$$

If by integrating with intermediate production – say, by merging with the firms producing 1 and 2 that become plants of the whole complex (then vertical integration is no more than an accounting device); or by actually replacing the whole process and rely on upstream primary factors rather than intermediate products -, the firm that produces y would face the function:

$$y = f[g^1(L^y{}_1), g^2(L^y{}_2), L_y] \tag{3}$$

there are no technological advantages nor disadvantages in joint (i.e., with x_1 and x_2 simultaneously) vertical integration.

Suppose we are assessing integration between the firms that produce y and intermediate input 1 only; technology constraints do not allow the output $y = f[g^1(L^y{}_1), x_2, L_y]$ with such arrangement; instead, the efficient production of y without the direct use of x_1 even if with the primary factor used in its production, L_1, is possible according to a production function

$$y = h^1(L^y{}_1, x_2, L_y) \tag{4}$$

Eventually, if $L^y{}_1$ and L_y refer to the same factor, $h^1(L^y{}_1, x_2, L_y) = h^1(x_2, L^y{}_1 + L_y) = h^1(x_2, L^{h1})$ and the distinction between $L^y{}_1$, say, assigned to the internal production of input 1, and L_y, other factor usage, becomes meaningless; without loss of generality, we will keep the distinction.

If $h^1(L^y{}_1, x_2, L_y) > f[g^1(L^y{}_1), x_2, L_y]$, there are technical *economies of depth* with respect to input 1. With perfect information and competitive markets, one would expect to observe only the vertically integrated arrangements. If $h^1(L^y{}_1, x_2, L_y) < f[g^1(L^y{}_1), x_2, L_y]$, there are *diseconomies of depth* - chain diseconomies, nesting diseconomies - with respect to input 1.

Economies of depth with respect to each input may interact in various ways. For example, there may be economies of depth with respect to 1 and 2 unilaterally, so that $h^1(L^y{}_1, x_2, L_y) > f[g^1(L^y{}_1), x_2, L_y]$ and $h^2(x_1, L^y{}_2, L_y) > f[x_1, g^2(L^y{}_2), L_y]$ - $h^2(x_1, L^y{}_2, L_y)$ represents the integrated technology with input 2 only, differing from $h^1(L^y{}_1, x_2, L_y)$ -, and yet the *joint* integration, $h(L^y_1, L^y_2, L_y)$ does not improve upon each of the unilateral arrangements, $h^1(L^y_1, x_2, L_y)$ or $h^2(x_1, L^y_2, L_y)$, even if it does the decentralized $f[g^1(L^y_1), g^2(L^y_2), L_y]$.

A dual representation may thus correspond to the previous definition. Let p_y denote the price of y, p_i the price of input i, i = 1,2, and w the price of a now homogeneous input L. The cost function of the generic producer using intermediate products – with quasi-concave technology in the three arguments, x_1, x_2 and L_y as (1) - is $C(y, p_1, p_2, w)$, originating conditional demands $x_i(y, p_1, p_2, w) = \partial C(y, p_1, p_2, w)/ \partial p_i$; of an intermediate product firm – with concave technology -, it is $C^i(x_i, w)$, with supply function $x_i(p_i, w)$ generated from $p_i = \partial C^i(x_i, w)/ \partial x_i$ [3]. From equilibrium between supplies and demands, $x_i(y, p_1, p_2, w) = x_i(p_i, w)$, i = 1,2, we derive a set of price equations, $p_i = p_i(y, w)$.

If intermediate products are manufactured by single producers – or by CRS technologies – and the intermediate market is competitive, integration will be expected – there will be economies of depth - iff

$$C^h(y, w) < C[y, C^1(x_1, w)/x_1, C^2(x_2, w)/x_2, w] < C[y, p_1(y,w), p_2(y,w), w] \quad (5)$$

where $C^h(y, w)$ denotes the cost function for the generic integrated production, say, $h(L^y_1, L^y_2, L_y)$, and the x_i's in (6) are evaluated at the unintegrated market optimal size level.

Partial economies of depth with respect to one input only, say 1, could as well be related to $C^{h1}(y, p_2, w) < C[y, C^1(x_1, w)/x_1, p_2, w] < C[y, p_1(y,w), p_2, w]$, where $C^{h1}(y, p_2, w)$ would be the cost function associated with (4).

Suppose that post-merger technology with both x_1 and x_2 allows $y = f[g^1(L^y_1), g^2(L^y_2), L_y, L^y_1, L^y_2]$ so that $f(x^y_1, x^y_2, L_y, 0, 0)$ identifies the decentralized production function of y from externally produced x_1 and x_2, i.e.:

[3] Production theory results and properties are commonly referred to in textbooks and surveys - Varian (1992), Diewert (1982), Nadiri (1982) and Chambers (1988) for example. Also, Silberberg (1990).

$$y = f(x^y_1, x^y_2, L_y, 0, 0) = f(x^y_1, x^y_2, L_y) \tag{6}$$

Then, economies of depth arise in the form of externalities - or rather, as *internalities* - implicit in valuation through $f(x^y_1, x^y_2, L_y, L^y_1, L^y_2)$ of the last two arguments, only present with vertical integration (they might as well be introduced as functions of the input quantities themselves…), with internal production of the intermediate products. Economies of depth with respect to input, say 1, would occur iff $f[g^1(L^y_1), x^y_2, L_y, L^y_1, L^y_2] > f[g^1(L^y_1), x^y_2, L_y, 0, L^y_2]$, to input 2, iff $f[x^y_1, g^2(L^y_2), L_y, L^y_1, L^y_2] > f[x^y_1, g^2(L^y_2), L_y, L^y_1, 0]$ - possible represented by $f_4[g^1(L^y_1), g^2(L^y_2), L_y, L^y_1, L^y_2]$, $f_5[g^1(L^y_1), g^2(L^y_2), L_y, L^y_1, L^y_2] > 0$ – positive "internalities" -, respectively; diseconomies would be associated to negative marginal effects.

Technically, presence of (dis)economies of depth defined in this way involve the collapse of the weak separability – even if arguments of $g^1(.)$ are priced equally in the market as those in $g^2(.)$, they are of independent usage in each of the subfunctions… - implicit in the form $f[g^1(L_1), g^2(L_2), L_y, 0, 0]$, even if separability between the input usage in the two sub-production functions is preserved with vertical integration.

Suppose further that the technology representing unilateral integration of y with firm 1 stems from the same 5-argument function as $f[g^1(L^y_1), x_2, L_y, L^y_1, 0]$, and that with firm 2 obeys $f[x_1, g^2(L^y_2), L_y, 0, L^y_2]$; then, (dis)economies of depth enjoy some sort of *separability* themselves. If we think of monetary, say cash-in-advance economies, natural – separable - economies of depth arise just for the fact that vertically integrated industries would save $r\ (p_1\ x_1 + p_2\ x_2 - w_1\ L_1 - w_2\ L_2)$[4], where r denotes the interest rate, each period of production just by being fully integrated.

Then, unilateral economies of scale with respect to input, say, 1 with the other kept external would occur iff $f[g^1(L^y_1), x^y_2, L_y, L^y_1, 0] > f[g^1(L^y_1), x^y_2, L_y, 0, 0]$, to input 2, iff $f[x^y_1, g^2(L^y_2), L_y, 0, L^y_2] > f[x^y_1, g^2(L^y_2), L_y, 0, 0]$ - possibly represented by $f_4(x^y_1, x^y_2, L_y, 0, 0)$, $f_5(x^y_1, x^y_2, L_y, 0, 0) > 0$,

[4] $r\ (p_1\ x_1 + p_2\ x_2 - w_1\ L_1 - w_2\ L_2 - p_{k1}\ \delta_1\ K_1 - p_{k2}\ \delta_2\ K_2)$ where p_{ki} denotes the unit price of physical capital, δ_i the capital stock depreciation rate and K_i the stock of capital in sector i if capital were also used in upstream firms and its owners "residual claimants" of the firms accounting profits, not requiring pre-money.

respectively; diseconomies would be associated to negative marginal effects.

Global economies of depth arise iff $f[g^1(L^y_1), x^y_2, L_y, L^y_1, L^y_2] > f[g^1(L^y_1), x^y_2, L_y, 0, 0]$. Then, they may occur even if unilaterally and keeping other input process out they may not (for example, a case in which $f_4 < 0$, $f_5 < 0$ but $f_{45} > 0$).

To assess the arising of a vertical integrated arrangement in such deterministic scenario, one would weight

1) Technological economies of depth with respect to each input.

2) The costs of internal production of the inputs relative to the cost at which it is available in the *market*. Then, scale of optimal demand of x^y_i by the firm that produces y relative to that of the typical firm producing i, returns to scale at the their production level, profits made by a firm in the intermediate product industry, and the competitive environment in the intermediate product industry are also relevant.

Proposition 1:

1.1. With CRS (or decreasing returns to scale, DRS, at the firm level, with smaller scale than downstream firms require) intermediate product technologies, and intermediate product industries operating at zero profits – facing competitive primary factor markets themselves -, unless we have economies of depth, no vertical integration is expected.

1.2. In the absence of (dis)economies of depth and if the firm producing y is the only user of the two inputs, provided the intermediate product firms behave competitively – i.e., price at marginal cost and face competitive factor markets -, market outcomes are invariant to vertical integration. As long as the intermediate product firms are making positive profits, integration is, however, to be desired by downstream producers – by internalizing production, these will accrue to the downstream firm (in addition to those already got…). And also the reverse…

The presence of a unique buyer-seller of each intermediate product has interesting implications for the understanding of the non-technological determinants of vertical integration. Under such circumstances, strategic behaviour of both buyer and sellers – the framework suggesting a model of two differentiated products competition - as market power of the seller of final product y, and of supplier of primary factor L are relevant in the determination of market structure.

Obviously, barriers to vertical integration – institutional, international or other - may exist. For simplicity, we will consider a single intermediate input economy and L_1 and L_y as quantities of an homogeneous primary factor in most of the sections – whenever we are not focusing on their behavior.

A general qualification of welfare effects can also be made. Obviously, given that we descend to primary factors in all production stages, the welfare loss of any vertical arrangement will depend on final output demand configuration, but also on primary factor supplies. The traditional surplus evaluation now suggests that:

- if both final demand and factor supplies are perfectly elastic, the welfare losses are confined to total industry profits waste.

- if final demand is perfectly elastic but factor supplies are perfectly inelastic, welfare losses will approach final revenue decreases and be in line with final output contraction in the downstream market.

- only if factor supplies are perfectly elastic can welfare losses be ascertained by the sum of consumer surplus and profits decreases. Or rather, if factor supplies are not perfectly elastic (nor perfectly inelastic), producer surplus measurement should internalize factor supply response.

3. Monopolistic upstream markets

3.1. Modelling vertical (or other) restraints

When assessing the possibility of an upstream firm exerting market power, one has to make assumptions on how it perceives the downstream firm reaction to its control variable – the downstream firm demand reaction to the price the upstream firm sets. That may be seen as constrained by the sequence of the decision process itself, or by the knowledge the upstream firm has by the time her own decision takes place on variables that the downstream firm either controls or faces. Or by the pre-committed level of these variables implied by particular transaction arrangements. Of course, the matter is not relevant for an upstream price-taker – she just assumes fixed output prices, invariant to her own scale decisions; it does otherwise.

Let $x_i(p_y, p_i, x_j, L_y)$ denote the short run demand for input i, arising from the solution of $p_y\ f_i(x_1, x_2, L_y) = p_i$. To identify it as its perceived demand, the upstream firm must, at the time of its production decision, observe all the four arguments in the function; and to use it for optimization, it must not observe p_j, the monopolistic competitor's price – in which case, it could use instead $x_i(p_y, p_1, p_2, L_y)$, arising from $p_y\ f_i(x_1, x_2, L_y) = p_i$ for both i = 1,2 -, nor w.

Or p_y, x_j and L_y are just fixed, while i's decisions concerning its price and supply take place. If labor contracts are of longer duration than – or settled before – those concerning the transaction of the upstream firm intermediate inputs - these decided at the same, and lower, hierarchic level in the decision process -, one would postulate that i recognizes only the short–run conditional demand $x_i(p_y, p_1, p_2, L_y)$. If labor could be adjusted also in the short run - yet an overall production scale y is (still) decided upon by downstream firms along with final output pricing prior to the factor and input-mix negotiation -, we would admit that the upstream firm faces the conditional demand:

$$x_i = x_i(y, p_1, p_2, w) \tag{7}$$

Of course, ex-post, y will be subject to optimization by the downstream firm. But at the time i makes decisions, and during the period in which the pertaining activities occur, it is seen as invariant [5].

We will concentrate on the comparison of two cases: one in which the conditional demand is identified – quantity-fixing, QF; another where the derived demand is – then, resale price maintenance, RPM, is suggested (or a perfectly elastic final output demand and no vertical restraints – theoretically, the optimizing behaviour of an upstream monopolist under a perfectly elastic final output demand in a perfectly informed, vertically unconstrained economy). We neglect the problem of firms multiplicity or entry dynamics in each (homogeneous) market – the setting may thus be seen to apply literally to single producer sectors (facing a perfectly elastic demand when their competitive selling behaviour is to be realistically staged); or to a competitive selling sector of firms with CRS technology – then, the firms/sector would always make nul profits.

Being the intermediate product firm i a monopolistic producer facing the conditional demand, it will set p_i such that it maximizes

$$p_i\, x_i(y, p_1, p_2, w) - C^i[x_i(y, p_1, p_2, w), w] \tag{8}$$

In a market where simultaneous downstream scale and hiring decisions take place – in which firm i links p_y to the demand it faces – the upstream firm would internalize the buyer *derived* demand $x_i(p_y, p_1, p_2, w)$ and maximize instead

$$\begin{aligned} p_i\, x_i(p_y, p_1, p_2, w) - C^i[x_i(p_y, p_1, p_2, w), w] &= \\ = p_i\, x_i[y(p_y, p_1, p_2, w), p_1, p_2, w] - C^i\{x_i[y(p_y, p_1, p_2, w), p_1, p_2, w], w\} & \end{aligned} \tag{9}$$

$y(p_y, p_1, p_2, w)$ denotes the downstream supply function.

In either case, optimization may be subject to conjectures on reaction of firm j to the price strategy of firm i, $p_j = s^j(p_i, y, w)$ or $p_j = s^j(p_i, p_y, w)$ respectively. FOC require:

[5] The setting reproduces a special two-stage game - see an example in Kreps and Scheinkman (1983), generating Cournot outcomes after Bertrand strategies in homogeneous product markets with capacity–induced quantity pre-commitment.

$$x_i(k, p_1, p_2, w) + \{p_i - \partial C^i[x_i(k, p_1, p_2, w), w]/ \partial x_i\}$$
$$[\partial x_i(k, p_1, p_2, w)/ \partial p_i + \partial x_i(k, p_1, p_2, w)/ \partial p_j\ \partial s^j(p_i, k, w)/ \partial p_i] = 0\ ,\ k = y, p_y$$

SOC for a maximum establish that the derivative of (the left hand-side of) the previous expression with respect to p_i must be negative around the solution obeying FOC.

Let i be a monopolist – and j sold in a large competitive market at fixed price p_j. $p_i - \partial C^i[x_i(k, p_1, p_2, w), w]/ \partial x_i$ reacts to p_i according to $1 - d\{\partial C^i[x_i(k, p_1, p_2, w), w]/ \partial x_i\}/dp_i = 1 - \partial^2 C^i[x_i(k, p_1, p_2, w), w]/ \partial x_i^2\ \partial x_i(k, p_1, p_2, w)/ \partial p_i$, positive if (but not only if) i works in the neighborhood of decreasing returns to scale (required by SOC for a price-taker); then, to create a (positive) wedge between p_i and marginal cost, the price should rise – and (at fixed p_j, k and w) p_i would be larger for a monopolist than for a price-taker.

(In our definition,) Final output pre-commitment implies that the downstream firm scale decision is outside the range of the input producer optimization – that y can be pre-ordered, insured, by the downstream firm without i's agreement which with input substitutability is a reasonable assumption. And the use of a particular demand form $x_i(.)$ in i's maximand also means that i is involved in such agreement: that she will supply the desired $x_i(., p_i, .)$ at the arguments' level of the function provided that p_i is guaranteed.

One could argue that short-run demands would more adequately apply to the scenario. Consider the extreme case that $x_i(y, x_2, L_y)$, solving $y = f(x_1, x_2, L_y)$ was to be used. Then, pre-commitment would mean that i always guarantees x_i to the downstream firm, which therefore, indeed fixes x_i and y (and chooses all else after and regardless of arrangements with i) – that i will provide x_i compatible with any y desired by the downstream firm, regardless of the p_i… I.e., i chooses y that maximizes $p_i\ x_i(y, x_2, L_y) - C^i[x_i(y, x_2, L_y), w]$: with downstream price-takers, the fully competitive solution is then achieved... In another angle, that would correspond to the vertical integration outcome.

If $x_i(p_y, p_i, .)$, then final price is maintained – invariant – during negotiations. It applies to bargaining agreements arranged through explicit resale price maintenance clauses; or under perfectly elastic final output demand with (some) internalization of final price (and then RPM is just implicit).

Obviously, the upstream firm will benefit from the removal of the quantity pre-commitment clauses, (or of the resale price maintenance clauses when final demand is less than perfectly elastic and the downstream market is competitive): if decisions on y are adjustable by the downstream firm *after* negotiations on x_i and p_i, i is in fact a "follower" after another - constrained to play a symmetric Nash-bargaining game - in what the decision on y is concerned. With the lifting, i can act as a "Stackelberg leader" – and cannot, therefore, be worse-off.

Proposition 2:

2.1. Monopolistic upstream firms can only deviate from competitive practices – and charge prices above marginal cost - if not all other input and factor purchases (or orders) were negotiated before – but totally adjustable by the downstream industry to – the upstream firm's price and quantity.

2.2. Under resale price maintenance schemes, one cannot have (active) quantity pre-commitment clauses – or vice-versa – affecting upstream optimization. That would not leave free the pre-determined levels of other products variables (either prices or quantities).

2.3. Obviously, the upstream firm will benefit from the removal of all pre-committed level if the latter was, in fact, pre-set by the downstream sector (and the removal does not change the downstream *modus operandi*, i.e., the perceived demand for the upstream intermediate product remains valid).

Likewise to 2.3, the upstream firm would not benefit from downstream firm short-run rigidity of other input or factor quantities during negotiations if these were decided optimally *ex-ante*.

3.2. Upstream monopoly with perfectly elastic final demand

Let y be pre-committed and i forced to a Nash equilibrium with respect to the downstream firm scale determination. Bertrand conjectures[6] - $\partial s^j(p_i, y, w)/\partial p_i = 0$ – or perfectly elastic supply of x_j and L_y at prices p_j and w to firm y imply that:

$$x_i(y, p_1, p_2, w) + \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\, \partial x_i(y, p_1, p_2, w)/\partial p_i = 0$$

or

$$- [1 - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i /p_i]\, \partial x_i(y, p_1, p_2, w)/\partial p_i\, p_i/x_i(y, p_1, p_2, w) = 1 \qquad (10)$$

[6] That is, Nash equilibrium in price strategies…

The conditional demand is negatively sloped and $\partial x_i(y, p_1, p_2, w)/ \partial p_i < 0$. Hence, in this case, - because $x_i(y, p_1, p_2, w) > 0$ - the intermediate firm(s) will set $p_i > \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i$.

(10) implies a response function $p_i = p^i(y, p_j, w)$. Of relevance is also how the price moves with $A = p_j$, y and w. On the one hand, the derivative of (10) with respect to p_i is negative for a maximum. Then at given other variables, the sign of $\partial p^i(p_j, y, w)/ \partial A$, $A = p_j$, y, w is that of the derivative of (10) with respect to $A = p_j$, y, w. For $A = p_j$, y:

$$\partial x_i(y, p_1, p_2, w)/ \partial A \; + \tag{11}$$
$$+ \; \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\} \, \partial^2 x_i(y, p_1, p_2, w)/(\partial p_i \, \partial A) \; -$$
$$- \; \partial^2 C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i^2 \; \partial x_i(y, p_1, p_2, w)/ \partial A \; \partial x_i(y, p_1, p_2, w)/ \partial p_i$$

The intermediate term can be replaced by:

$$- \; \partial^2 x_i(y, p_1, p_2, w)/(\partial p_i \, \partial A) \, x_i(y, p_1, p_2, w) \, / \, \partial x_i(y, p_1, p_2, w)/ \partial p_i \tag{12}$$

As long as this is negligible (say, for prices around marginal cost) or favorable (e.g., along with the first term), expression (11) will have the sign of $\partial x_i(y, p_1, p_2, w)/ \partial A$. For A = y, being i a normal input in the downstream technology – i.e., $\partial x_i(y, p_1, p_2, w)/ \partial y > 0$ -, it will be positive; being regressive, it will be negative. For $A = p_j$, it has the sign of $\partial x_i(y, p_1, p_2, w)/ \partial p_j$, positive if i and j are substitutes in production in the downstream technology, negative if they are complements.

For A = w, (11) includes also the term

$$- \, \partial^2 C^i[x_i(y, p_1, p_2, w), w]/(\partial x_i \, \partial w) \, \partial x_i(y, p_1, p_2, w)/ \partial p_i \tag{13}$$

having the sign of $\partial^2 C^i[x_i(y, p_1, p_2, w), w]/(\partial x_i \, \partial w) = \partial L^i[x_i(y, p_1, p_2, w), w]/ \partial x_i$, of how the "competitive" conditional demand for L_i by firm i responds to its own scale. If L_i was priced independently of L_y, (13) would not be included along with (11) in the measurement of the effect of the price of L_y, and it would be the only term relevant in the measurement of the effect of the price of L_i – positive (negative) if L_i is normal (regressive) in the upstream production process [7].

Being i and L_y substitutes (in production) in y - $\partial x_i(y, p_1, p_2, w)/\partial w > 0$ -, L_i a normal factor for firm i, and (12) negligible, (11) plus (13) will be positive.

I's profit will change with A = y, p_j according to $\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\ \partial x_i(y, p_1, p_2, w)/\partial A$, and with w as $\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\ \partial x_i(y, p_1, p_2, w)/\partial w - L_i[x_i(y, p_1, p_2, w), w]$, where $L_i[x_i(y, p_1, p_2, w), w]$ denotes the conditional demand of firm i.

Proposition 3:

Under quantity pre-commitment, exogenous to upstream monopoly determination:

3.1. Monopolistically competitive upstream firms will set prices above marginal cost. For a given final output level, this will necessarily imply a shift to the use of primary factor (if there are no other inputs) in downstream production – and a total inefficiency by excessive hiring in the whole industry – relative to the price-taker equilibrium.

3.2. A rise in scale will imply a rise (decrease) in the monopolist price, provided the input is normal (regressive) for firm y. A rise in other intermediate product price will raise (decrease) the monopolist price if the inputs are substitutes (complements) in the downstream technology. A rise in wages (primary factor prices) will most likely imply a rise in prices if input is substitute (complements) to labor in downstream technology and normal (regressive) in the upstream technology.

3.3. A rise in scale will imply a rise in the upstream monopolist profits, provided the input is normal for firm y. A rise in other intermediate product price will raise (decrease) the monopolist profit if the inputs are substitutes (complements) in the downstream technology. A rise in wages (primary factor prices) will imply a decrease in the monopolist profit if (but not only if) the input is complement to labor in downstream technology.

Even if y is pre-arranged to upstream negotiations – provided the downstream firm is competitive in the final output market and allowed to maximize its profits - equilibrium final output will ultimately be replaced in (10). If final output demand is perfectly elastic at price p_y, that amounts to say that downstream supply, $y(p_y, p_1, p_2, w)$ is replaced in (10) - which implies that $p_i = p^i[p_j, y(p_y, p_1, p_2, w), w]$ will respond to p_y in the same direction as to y. (Note that, using Hotelling's lemma, $\partial p^i[p_j, y(p_y, p_1, p_2,$

[7] With only one input in upstream technology, it will be normal.

w), w]/ ∂y $\partial y(p_y, p_1, p_2, w)/\partial p_i = -\partial p^i[p_j, y(p_y, p_1, p_2, w), w]/\partial y$ $\partial x_i(p_y, p_1, p_2, w)/\partial p_y < 1$ always, once the left hand-side is always negative.)

As $y(p_y, p_1, p_2, w)$ answers to p_y – and input prices -, the further internalization of the downstream firm output scale optimization process also affects the equilibrium outcome. Additional reaction of supply of final product would lead to

$$x_i(p_y, p_1, p_2, w) + \{p_i - \partial C^i[x_i(p_y, p_1, p_2, w), w]/\partial x_i\}\, \partial x_i(p_y, p_1, p_2, w)/\partial p_i = 0 \tag{14}$$

which, because $x_i(p_y, p_1, p_2, w) = x_i[y(p_y, p_1, p_2, w), p_1, p_2, w]$, in fact adds to (10) the term:

$$\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\} \circledcirc x_i(y, p_1, p_2, w)/\partial y\, \partial y(p_y, p_1, p_2, w)/\partial p_i \tag{15}$$

It is clear that the solution of (10) evaluated at the final demand $y(p_y, p_1, p_2, w)$, expected, say, if y is pre-committed, differs from the new one, which would also include (15). Obviously, at given (p_y, p_j, w) – if final demand is perfectly elastic -, i is always better-off without (any) pre-commitment. In practice, RPM could then be a device to escape quantity fixing - conditional demand bargaining - and/or a means to achieve a better understanding of the downstream sector optimization process.

Duality theory proves that $\partial x_i(y, p_1, p_2, w)/\partial y\, \partial y(p_y, p_1, p_2, w)/\partial p_i < 0$ always – it measures the scale effect on the demand for x_i of a rise in its price, p_i. Therefore, we expect that a further internalization of scale optimization (at given w and p_j) decreases p_i (lowers i's reaction function) and, therefore, will tend to increase x_i. Also, QF before could take the form of a ceiling (floor), of a maximum (minimum) output-scale-capacity clause if i is normal (regressive): as y responds according to downstream supply $y(p_y, p_1, p_2, w)$, it would be set at a higher (lower) level without the restraint.

Otherwise, conditions involving conditional demands are now replaced by others relating to derived demand. For instance, $\partial x_i(p_y, p_1, p_2, w)/\partial A$, $A = p_y, p_j$, still conditions the effect of A on firm's i reaction function. For $A = p_j$, it is now substitutability at the derived demands that affect the slopes of the reaction functions. For $A = p_y$, as $\partial x_i(p_y, p_1, p_2, w)/\partial p_y = \partial x_i(y, p_1, p_2, w)/\partial y\, \partial y(p_y, p_1, p_2, w)/\partial p_y$, factor normality still conditions the sign effect.

For a fixed final output price, an upstream monopoly would still fix a higher intermediate input price than a price-taker. At p_j and w – for i monopolist -, the optimal profits increase (decrease) with p_y if i is normal (regressive) in downstream technology because then $\{p_i - \partial C^i[x_i(p_y, p_1, p_2, w), w]/\partial x_i\}\, \partial x_i(p_y, p_1, p_2, w)/\partial p_y > (<) 0$.

Downstream firm's profits rise with A iff $\partial P(p_y, p_1, p_2, w)/\partial A + \partial P(p_y, p_1, p_2, w)/\partial p_i\, \partial p^i/\partial A = \partial P(p_y, p_1, p_2, w)/\partial A - x_i(p_y, p_1, p_2, w)\, \partial p^i/\partial A > 0$, $A = p_y, p_j, w$, where $P(p_y, p_1, p_2, w)$ denotes y's conventional profit function. They rise with p_y iff $y(p_y, p_1, p_2, w) - x_i(p_y, p_1, p_2, w)\, \partial p^i/\partial p_y > 0$, i.e., if average product $y(p_y, p_1, p_2, w)/ x_i(p_y, p_1, p_2, w)$ is larger than $\partial p^i/\partial p_y$; they decrease with $A = p_j$, w if but not only if $\partial p^i/\partial A > 0$.

Proposition 4:

With monopolistic upstream firms and perfectly elastic final output demand – or under a fixed final output price:

4.1. Input price flexibility to downstream output price will lower upstream prices. It will thus raise (decrease) final output - provided the intermediate input is normal (regressive); and it will enhance both downstream and upstream firms profits.

4.2. Reaction to final output price has the same sign as to downstream firm scale. The sign responses to other intermediate input prices and wages remain valid relative to 3.2 and 3.3. but conditioned by substitutability in derived demand.

4.3. Vertical integration, insuring competitive upstream practices, will necessarily increase aggregate profits and welfare. It will increase (decrease) output provided i is normal (regressive) in downstream technology [8].

3.3. Upstream monopoly and interaction with final output demand and competitive environment

We have (implicitly) assumed a perfectly elastic final output demand. If we drop the assumption, the previous analysis becomes insufficient to qualify equilibrium. On the one hand, even if the downstream firm behaves competitively:

- the quantity pre-commitment solution (10) will hold and can be seen as evaluated at supply $y(p_y, p_1, p_2, w)$ but this suffers feedback – through p_y - from the final output market and demand.

- solution (14), or (10)+(15), will hold only if resale price maintenance clauses are present. If they are, they can be seen as evaluated at supply

[8] This is implicit from the next subsection conclusions.

$y(p_y, p_1, p_2, w)$ but, again, this suffers feedback from the final output market. Particularly, it is no longer true that (14) is better than (10) for an upstream monopolist – (14) does not insure against input demand fall feedback…

- without resale price maintenance or other restrictions – and with downstream always being a standard price-taker "follower" -, solution (14) will no longer hold: then, it can also benefit from its implicit monopoly position towards the final output market, as the sole supplier of an input all downstream sellers – even if many and competitive –require, that is always incorporated in the final product. (Of course, the upstream firm will be better-off under this arrangement.)

On the other hand, a downstream monopoly behaviour towards the final demand will interact differently with the three types of contracts or decision processes.

Let $p_y = p_y(y)$ denote the negatively sloped inverse demand for the final product y, $y = y(p_y)$ being the direct function. A downstream price-taker - with cost function $C^y(y, p_1, p_2, w)$ - behaviour will insure $y = y^C(p_1, p_2, w)$ such that satisfy:

$$p_y(y) = \partial C^y(y, p_1, p_2, w) / \partial y \quad \text{or} \quad y = y[p_y(y), p_1, p_2, w] \tag{16}$$

Then:

$$\partial y^C(p_1, p_2, w)/\partial A = \partial y(p_y, p_1, p_2, w)/\partial A \,/[1 - \partial y(p_y, p_1, p_2, w)/\partial p_y \; p_y(y)'] = \partial^2 C^y(y, p_1, p_2, w)/(\partial y\, \partial A) \,/\, [p_y(y)' - \partial^2 C^y(y, p_1, p_2, w) / \partial y^2]\,, \; A = p_1, p_2, w \tag{17}$$

It has the same sign as, but lower absolute value than $\partial y(p_y, p_1, p_2, w)/\partial A$, and opposite sign to $\partial^2 C^y(y, p_1, p_2, w)/(\partial y\, \partial A)$, for $\partial^2 C^y(y, p_1, p_2, w) / \partial y^2 > 0$, increasing marginal cost.

Supply and derived demands – being the downstream firm a price-taker towards inputs – will follow as functions of p_y, but now this is extraneously fixed at $p_y(y^*) = p_y[y^C(p_1, p_2, w)] = p_y^C(p_1, p_2, w)$ that solves (16).

The first thing to notice is that downstream (price-taker) firm optimal profits $P\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\}$ will decrease (increase) with an upstream price rise – say, due to the monopolization of upstream market - iff

$\partial x_i(p_y, p_1, p_2, w)/\partial p_y\ p_y / x_i < (>) - \partial y(p_y)/\partial p_y\ p_y / y\ [1 - p_y(y)'\ \partial y(p_y, p_1, p_2, w)/\partial p_y] = - \partial y(p_y)/\partial p_y\ p_y / y + \partial y(p_y, p_1, p_2, w)/\partial p_y\ p_y / y$

$$\partial y(p_y, p_1, p_2, w)/\partial p_y\ p_y/y\ [\partial x_i(y, p_1, p_2, w)/\partial y\ y/x_i - 1] < (>) - \partial y(p_y)/\partial p_y\ p_y/y \tag{18}$$

i.e., iff final demand is very elastic (inelastic). (Because it reacts to a change dp_i as $[\partial\ (p_y, p_1, p_2, w)/\ \partial p_y\ p_y(y)'\ \partial y^C(p_1, p_2, w)/\ \partial p_i + \partial P(p_y, p_1, p_2, w)/\ \partial p_i]\ dp_i = [-\ y(p_y, p_1, p_2, w)\ p_y(y)'\ \partial x_i(p_y, p_1, p_2, w)/\ \partial p_y\ /\ [1 - p_y(y)'\ \partial y(p_y, p_1, p_2, w)/\ \partial p_y] - x_i(p_y, p_1, p_2, w)\}\ dp_i$.) When sector y, that prices at marginal cost, benefits with such rise (decrease) in a normal (regressive) intermediate input price, it is as if upstream cartelization, for example, or the vertical restraint, pushed the final market towards a solution closer to its own monopolization; obviously, if the downstream market is constrained to behave competitively, vertical integration could then harm aggregate profits - it always benefits aggregate welfare.

It is immediate to conclude that under pre-commitment (10) will lead to a decrease (increase) in output – and, for (16) to hold, require an increase in input prices – after monopolization of the market of a normal (regressive) input, say, input 1. (16) establishes $p_y(y) - \partial C^y(y, p_1, p_2, w) / \partial y = g(y, p_1, p_2, w) = 0$, $\partial g/\partial y < 0$ (for SOC to hold) and $\partial g/\partial p_1 < 0$ if 1 is normal. At y^*, p_1^* of perfect competition, $g(y^*, p_1^*, p_2, w) = 0$. Using (10), we conclude that $p_1(y^*, w) > p_1^*$ once the latter is fixed at marginal cost of firm 1; that is, $g[y^*, p_1(y^*, w), p_2, w] < 0$ and y has to "decrease" to restore equality. Of course, intermediate input prices ("do not decrease" because they were not set at $p_1(y^*, w)$…) must now be higher because $g(y, p_1, p_2, w) = 0$ must still hold and y is now lower. p_y – because final demand is negatively sloped - is necessarily higher.

Alternatively, if 1 is normal, $y = y^C(p_1, p_2, w)$ is negatively sloped and (10) generates a positively sloped function $p_1 = p_1(y, w)$ in space (y, p_1). $p_1(y, w)$ is necessarily above the competitive locus $p_1 = \partial C^1[x_1(y, p_1, p_2, w), w]/\ \partial x_1$. Therefore, the upstream monopoly solution E – see Fig. 1 - must be to the northwest of the competitive solution C.

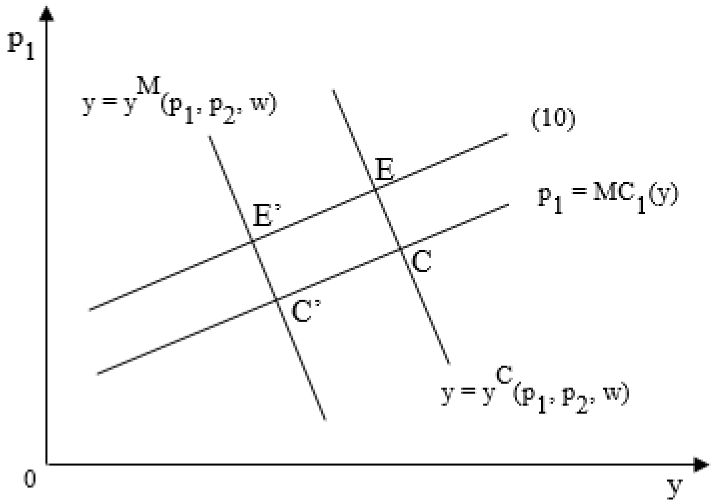

Figure 1. *Quantity Pre-Commitment - 1 normal*

If 1 is regressive – Fig. 2 – E will lie to the northeast of C – implying a higher intermediate input price and final output level under upstream cartelization.

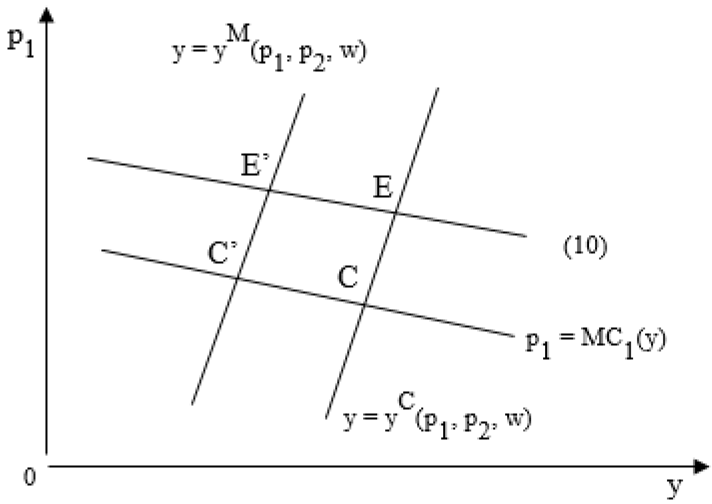

Figure 2. *Quantity Pre-Commitment - 1 regressive*

Under RPM, we could attempt to develop the same reasoning but with respect to p_y. If 1 is normal, (14) generates a positively sloped relation $p_1 = p_1(p_y, w)$ in space (p_y, p_1), but also does $p_y = p_y[y^C(p_1, p_2, w)]$: the assessment of the effect of upstream cartelization becomes dependent on the relative size of the slopes of the two curves. If $p_y = p_y[y^C(p_1, p_2, w)]$ has higher slope – say, if final output demand is very elastic -, cartelization raises both the intermediate and final output price; otherwise, it decreases them.

With no vertical restraints:

$x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\} +(p_i - \partial C^i[x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\}, w]/ \partial x_i) [\partial x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\}/\partial p_i + \partial x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\}/\partial p_y\, p_y[y^C(p_1, p_2, w)]'\, \partial y^C(p_1, p_2, w)/ \partial p_i] = 0$ or

$x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\} +(p_i - \partial C^i[x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\}, w]/ \partial x_i) [\partial x_i\{p_y[y^C(p_1, p_2, w)], p_1, p_2, w\}/\partial p_i - \partial x_i[y^C(p_1, p_2, w), p_1, p_2, w]/ \partial p_i\, p_y[y^C(p_1, p_2, w)]'\, \partial y(p_y, p_1, p_2, w)/ \partial p_y] / [1 - \partial y(p_y, p_1, p_2, w)/ \partial p_y\, p_y(y)'] = 0$ (19)

As long as SOC hold, it can again be reasserted that the equilibrium intermediate input price under upstream cartelization should be higher than the competitive one. Then – due to (16) -, output decreases (rises) with it provided i is normal (regressive) in downstream technology.

The lifting of resale price clauses – generating (19) - will imply that (14) will be added of

$\{p_i - \partial C^i[x_i(p_y, p_1, p_2, w), w]/ \partial x_i\}\, p_y[y^C(p_1, p_2, w)]'\, \partial y^C(p_1, p_2, w)/ \partial p_i = \{p_i - \partial C^i[x_i(p_y, p_1, p_2, w), w]/ \partial x_i\}\, p_y[y^C(p_1, p_2, w)]'\, \partial y(p_y, p_1, p_2, w)/ \partial p_i /[1 - \partial y(p_y, p_1, p_2, w)/ \partial p_y\, p_y(y)'] = \{p_i - \partial C^i[x_i(p_y, p_1, p_2, w), w]/ \partial x_i\}\, p_y[y^C(p_1, p_2, w)]'\, \partial x_i(y, p_1, p_2, w)/ \partial y / [p_y(y)' - \partial C^2(y, p_1, p_2, w)/ \partial y^2]$ (20)

(20) will be positive (negative) if i is normal (regressive): normal (regressive) intermediate input prices will rise (decrease) with the lifting of RPM. Because of (16), $y^C(p_1, p_2, w)$ – and $p_y{}^C(p_1, p_2, w)$ - holding with or without RPM, output will decrease and final output price will increase with the lifting of RPM (which may, therefore, take here the form of a ceiling, of a maximum output price clause).

With the lifting of quantity pre-commitment, the upstream firm will look for (10) plus:

$\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\}\, \partial y^C(p_1, p_2, w)/ \partial p_i = \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\}\, \partial x_i(y, p_1, p_2, w)/ \partial y / [p_y(y)' - \partial C^2(y, p_1, p_2, w)/ \partial y^2]$ (21)

(10) + (21) must decrease with p_i for SOC to hold. If i is normal (regressive), (21) is negative (positive); then, at the solution satisfying (10),

(10) + (21) < (>) 0: p_i must decrease (increase) to restore equality to 0 - intermediate prices decrease (increase) without quantity pre-commitment.

Proposition 5:

With monopolistic upstream firms, not perfectly elastic final output demand and competitive downstream sector:

5.1. With quantity pre-commitment or no vertical restraints, intermediate input cartelization raises its price above the competitive level. With RPM, a sufficiently elastic demand would produce the same result – which, in general, is no longer guaranteed. If it occurs, if the input is normal (regressive) in downstream technology, final output produced will suffer (increase) with upstream cartelization.

5.2. Resale price maintenance will decrease (increase) normal (regressive) intermediate input prices and raise final output.

5.3. Quantity pre-commitment will increase (decrease) normal (regressive) intermediate input prices and decrease final output.

5.4. Vertical restraints will harm upstream firms.

5.5. Downstream firms will benefit from intermediate input price decreases (rises) – due to cartelization of upstream markets, vertical restraints or other - iff (18) holds (does not hold).

5.6. Vertical integration may decrease aggregate profits. It will always raise total welfare.

One can further qualify the additional effect of a downstream monopoly equilibrium [9] – a possibly more realistic hypothesis, even if a downstream firm can sell in an international market and still face local input markets. The downstream firm - price-taker with respect to the input markets - will set y such that $y = y^M(p_1, p_2, w)$:

$$p_y(y) + y\, p_y(y)' = \partial C^y(y, p_1, p_2, w) / \partial y \tag{22}$$

At the same input prices, the monopoly "derived" or "reduced" supply will be smaller than the competitive one: $y^M(p_1, p_2, w) < y[p_y(y^C), p_1, p_2, w] = y^C(p_1, p_2, w)$ derived under (16). Now, from (22)

$$\partial y^M(p_1, p_2, w)/ \partial p_i = \partial x_i(y, p_1, p_2, w)/ \partial y \,/$$
$$/\, [2\, p_y(y)' + y\, p_y(y)'' - \partial C^2(y, p_1, p_2, w)/ \partial y^2] \tag{23}$$

[9] The chain of monopolies was early assessed by Spengler (1950). Upstream and downstream oligopolist equilibria were studied by Greenhut & Ohta (1979). In their industry, upstream firms have CRS technologies and downstream firms are pure intermediaries. They conclude that final output increases - and final price declines – with vertical integration of Cournot agents.

Any feedback through the downstream market will have the same sign as with a competitive downstreamer, but attenuated iff $p_y(y)' + y\, p_y(y)'' <$ 0, i.e., – $y\, p(y)'' / p(y)' < 1$ (the relative measure of risk aversion of the inverse final demand is smaller than 1) – that is, when $p_y(y)$ (or $y(p_y)$ because it is negatively sloped) is not too convex.

Aggregate profits are obviously maximized if (22) holds and sector i (upstream sectors…) behaves competitively, i.e., charges at marginal cost $p_i = \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i$: one would choose y, p_i and x_i maximizing $p_y(y)\, y - C^y(y, p_1, p_2, w) + p_i\, x_i - C^i(x_i, w)$. That allocation is the one achieved through vertical integration [10] if downstream monopoly is insured.

Downstream (monopoly) profits – derived under (22) - will always decrease with an upstream price increase – say, due to the monopolization of upstream market or QF.

As $y^M(p_1, p_2, w) < y^C(p_1, p_2, w)$, under quantity pre-commitment (10) – regardless of the feedback from the upstream market, equally present as $p_i(y, w)$ over (23) as if the term $y\, p_y(y)'$ was absent on it –, final output will be lower and then, - due to proposition 3.2 - equilibrium prices of a normal (regressive) input would be lower (higher) when the downstream firm is a monopoly than when it behaves competitively: we contrast E and E' in Fig. 1 (2). As final output is lower, if i is normal (regressive) i's profits are also lower (higher): i loses with (benefits from) downstream cartelization. The downstream firm benefits with its monopolization if (but not only if) i is normal – on the one hand, from its own downsizing at given input prices, on the other, from the induced input price reduction, occurring if i is normal.

Alternatively: at $y^M(p_1, p_2, w)$ and/of competitive price p_i^C, if i is normal (regressive), (10) is negative (positive). As $\partial^2 P^i(y, p_1, p_2, w)/ \partial p_i^2 + \partial^2 P^i(y, p_1, p_2, w)/(\partial p_i\, \partial y)\, \partial y^M(p_1, p_2, w)/ \partial p_i < 0$ provided (12) is negligible (because then $\partial^2 P^i(y, p_1, p_2, w)/(\partial p_i\, \partial y) > (<) 0$ if y is normal (regressive) and then $\partial y^M(p_1, p_2, w)/ \partial p_i < (>) 0$, the second term is always negative – the first must be for SOC to hold), the gap created in (10) closes with the fall (increase) of p_i.

[10] As noted in Westfield (1981), p. 338, for example. He assesses the result of a previously downstream competitive market rendered a monopoly towards final output demand after vertical integration.

Let us consider resale price maintenance clauses. If the downstream firm remains passive – price-taker - towards the upstream market, $y = y^M(p_1, p_2, w)$ would appear to remain valid; yet, derived demand faced by i is $x_i[y(p_y, p_1, p_2, w), p_1, p_2, w]$, with p_y seen as fixed, which would be incompatible with it. RPM is a conduct norm towards the final product market applied to downstream firms during intermediate input negotiations – price is seen as fixed and i always faces the competitive derived demand $x_i[y(p_y, p_1, p_2, w), p_1, p_2, w]$ – (14) always holds. Therefore, at choosing scale, either the downstream firm sees output price fixed and a perfectly elastic demand at the agreed price – and RPM leads to the invariance of market outcome relative to downstream competitive behaviour towards the final output market.

Or the optimal reduced supply differs from the previous ones and obeys $y = y^{M'}(p_1, p_2, w)$ such that:

$$y + y[p_y(y)]'\, p_y(y) = \partial C^y\{y[p_y(y), p_1, p_2, w], p_1, p_2, w\}/\partial y\; \partial y[p_y(y), p_1, p_2, w]/\partial p_y \quad \text{or}$$

$$p_y(y)'\, y + p_y(y) = \partial C^y\{y[p_y(y), p_1, p_2, w], p_1, p_2, w\}/\partial y\; \partial y[p_y(y), p_1, p_2, w]/\partial p_y\, p_y(y)' \tag{24}$$

while without them, it followed (22). Because $p_y(y)' < 0$, $p_y^{M'}$ must be smaller than p_y^M and $y^{M'}(p_1, p_2, w) > y^M(p_1, p_2, w)$. If $p_y(y)'\{\partial C^y(y, p_1, p_2, w)/\partial y\; \partial y[p_y(y), p_1, p_2, w]/\partial p_y - y\} > \partial C^y(y, p_1, p_2, w)/\partial y$ – if $y > \partial C^y(y, p_1, p_2, w)/\partial y\; \{\partial y[p_y(y), p_1, p_2, w]/\partial p_y - y(p_y)'\}$ -, it is still the case that $y^{M'}(p_1, p_2, w) < y^C(p_1, p_2, w)$ and $p_y[y^{M'}(p_1, p_2, w)] > p_y[y^C(p_1, p_2, w)]$. Then, if i is normal, equilibrium p_i with downstream monopoly will be higher (lower) than with perfect competition towards final demand iff in (14) $\partial^2 P^i(p_y, p_1, p_2, w)/\partial p_i^2 + \partial^2 P^i(p_y, p_1, p_2, w)/(\partial p_i\, \partial p_y)\; p_y(y)'\; \partial y^{M'}(p_1, p_2, w)/\partial p_i > (<) 0$… Now, under that same condition, with monopoly in downstream markets, (14) - which is smaller than (10) and therefore negative at the equilibrium levels of quantity pre-commitment – resale price maintenance will lead to higher (lower) intermediate price than quantity pre-commitment. But it can also happen that $y^{M'}(p_1, p_2, w) > y^C(p_1, p_2, w)$…

Finally, consider there are no vertical restraints. The first difference relative to the downstream price-taker case is that derived demand $x_i(p_y, p_1, p_2, w)$ loses its role completely: for example, the lifting of RPM no longer provides the internalization of a restriction for the upstream firm. In fact, with a downstream price-taker, the upstream firm will look for (10) plus (21) while the one facing monopoly will choose (10) plus:

$$\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\}\ \partial y^M(p_1, p_2, w)/ \partial p_i = \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\}\ \partial x_i(y, p_1, p_2, w)/ \partial y\ /\ [2\ p_y(y)' + y\ p_y(y)'' - \partial C^2(y, p_1, p_2, w)/ \partial y^2] \quad (25)$$

On the one hand, $y^M(p_1, p_2, w) < y^C(p_1, p_2, w)$, suggesting that if i is normal (regressive), (10) will be negative (positive) at the new *induced* price level. On the other, (25) is negative (positive) if i is normal (regressive) – and less negative (positive) than (21) if final demand is concave or not too convex – if $p_y(y)' + y\ p_y(y)'' < 0$. Then, if (but not only if) demand is not very convex, (10)+(21) is negative (positive) at $y^M(p_1, p_2, w)$ and p_i (p_i^C has to decrease (increase)) increases (decreases) with downstream cartelization. But even if demand is very convex, the first effect can still dominate…

We can also compare the two vertical arrangements with the new case. It is still true that the comparisons with QF remain valid – (25) is negative. With RPM under downstream monopoly, (10) is added of

$$\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\}\ \partial x_i(y, p_1, p_2, w)/ \partial y\ \partial y(p_y, p_1, p_2, w)/ \partial p_y\ p_y(y)'\ \partial y^{M'}(p_1, p_2, w)/ \partial p_i \quad (26)$$

while with no vertical restraints it was added of (25). If i is normal, (25) is negative, but, if $\partial y^{M'}(p_1, p_2, w)/ \partial p_i < 0$ – a likely result -, (26) is positive. Then, (10)+(25) is negative at the RPM price solution, which therefore implies lower intermediate prices in the absence of vertical restraints than with RPM. The opposite occurs if i is regressive.

Proposition 6:

With monopolistic upstream firms, elastic final output demand and a cartelized downstream sector:

6.1. Regardless of the existence of vertical restraints, demand feedback still allows intermediate input cartelization to raise its price above the competitive level.

6.2. Under quantity pre-commitment, the downstream industry cartelization will necessarily decrease final output, raise final output price, and if i is normal (regressive) lower (raise) intermediate input prices and upstream profits.

6.3. Under resale price maintenance, the downstream industry cartelization may raise final output price, decrease final output and, being i normal, increase intermediate input prices and upstream profits.

6.4. In absence of vertical restraint clauses, downstream cartelization will raise normal (decrease regressive) intermediate input prices if (but not only if) final demand is not very convex.

6.5. Resale price maintenance will (likely) increase normal (decrease regressive) intermediate input prices. It may not harm upstream firms.

6.6. Quantity pre-commitment will increase normal (decrease regressive) intermediate input prices. It will harm (benefit) upstream firms if i is normal (regressive).

6.7. Downstream firms will benefit – and final output will increase if i is normal (decrease if i is regressive) - from intermediate input price decreases – due to cartelization of upstream markets, vertical restraints or other – except if RPM is introduced.

6.8. Vertical integration, insuring implicit competitive upstream practices, will necessarily increase aggregate profits. It may be welfare detrimental.

3.4. Upstream monopoly and two-stage Bargaining under quantity pre-commitment

Quantity (or price) pre-commitment as previously modelled implied that final output (or price) was subject to downstream firm discretion. For upstream firms to be able to benefit from it, the downstream firms must have less autonomy towards the final output market than a standard price-taker has. Which – as it faces a monopoly intermediate price seller – may seem intuitively acceptable.

If there is only one intermediate input and ex-post, y always reacts according to its supply $y(p_y, p_1, w)$ – similarly to a conventional Stackelberg follower, as was assumed –, 1 benefits from internalizing it and prefers – as y does, once p_1 is lower… - unconstrained bargaining. More plausibly, with quantity pre-commitment, y would bargain on his short-run supply, $y = y(p_y, x_1, w)$; but, because the short-run supply evaluated at conditional derived demand $y = y[p_y, x_1(y, p_1, w), w]$ must solve for the long-run supply $y = y(p_y, p_1, w)$ - and derived demand is already embedded in the game -, we would not reach different conclusions.

That is, downstream quantity determination must also be under 1's discretion for her to benefit from pre-commitment. Two-stage games can be constructed to reproduce the double bargaining [11]:

1) As 1 benefits from a rise in y, a plausible outcome would be for 1 to force the maximum y that just warrants minimal profits to the downstream firm: $p_y\, y = C(y, x_1, w)$ or $p_y\, y = C(y, p_1, w)$: impose it instead of supply.

2) Say, for a monopolist, (10) generates a contingent (sub-game) equilibrium price, $p_1 = p_1(y, w)$, resulting from a first auction by downstream producers. Then, in an encompassing stage of the game, firm 1 sets x_1, maximizing $p_1[y(p_y, x_1, w), w]\, x_1 - C^1(x_1, w)$, where $y(p_y, x_1, w)$ denotes the short-run supply of firm y; it generates a response $x_1 = x_1(p_y, w)$ obeying:

$$p_1[y(p_y, x_1, w), w] + x_1\, \partial p_1[y(p_y, x_1, w), w]/\, \partial y\, \partial y(p_y, x_1, w)/\, \partial x_1 - \partial C^1(x_1, w)/\, \partial x_1 = 0$$

Then and ex-post quantity of the sub-game may either be:

- $x_1(p_y, w)$; $y = y[p_y, x_1(p_y, w), w]$; $p_1 = p_1\{y[p_y, x_1(p_y, w), w], w\}$

- $y = y[p_y, x_1(p_y, w), w]$; $x_1\{y[p_y, x_1(p_y, w), w], p_1(y[p_y, x_1(p_y, w), w], w), w\}$ and $p_1 = p_1\{y[p_y, x_1(p_y, w), w], w\}$. Then, the second-stage game just signals a short-run supply argument.

- $x_1(p_y, w) = x_1[y, p_1(y, w), w]$. From it, $y = y(p_y, w)$ that would solve the sub-game but that would be incompatible with (differ from) short-run supply $y[p_y, x_1(p_y, w), w]$. Then, $p_1 = p_1[y(p_y, w), w]$ – which will differ from $p_1\{y[p_y, x_1(p_y, w), w], w\}$...

In the first two cases, $x_1(p_y, w)$ is instrumental to fix the supply quantity y imputed into the price subgame solution, yielding $p_1 = p_1\{y[p_y, x_1(p_y, w), w], w\}$: we do not expect therefore that $x_1(p_y, w) = x_1[y, p_1(y, w), w]$… Also, the first is always better for 1 than the second.

Ex-post, p_1 is set below marginal cost only in the first case.

3) Say, for a monopolist, (10) generates a contingent (sub-game) equilibrium order, $x_1 = x_1[y, p_1(y, w), w] = x_1(y, w)$ placed by downstream producers. The second bargaining round is quantity-constrained, so that x_1 is fixed at that level; firm 1 sets p_1, maximizing $p_1\, x_1[y(p_y, p_1, w), w] -$

[11] We are, of course, assuming that the upstream firm is constrained to linear or proportional pricing. Otherwise, as previously pointed out in the literature – being the final demand perfectly elastic; or with downstream monopoly as illustrated in Tirole (1988), p. 176 –, the upstream firm would maximize results charging at marginal cost and appropriating the "autonomous" downstream firm economic profits through a lump-sum fee…

$C^1\{x_1[y(p_y, p_1, w), w], w\}$, where $y(p_y, p_1, w)$ denotes supply, idealized supply, of firm y; 1 generates a response obeying

$$x_1[y(p_y, p_1, w), w] + [p_1 - \partial C^1\{x_1[y(p_y, p_1, w), w], w\}/\partial x_1]\ \partial x_1[y(p_y, p_1, w), w]/\ \partial y\ \partial y(p_y, p_1, w)/\ \partial p_1 = 0$$

a solution $p_1 = p_1(p_y, w)$ and implies $y = y[p_y, p_1(p_y, w), w]$. The price now set may be:

- in fact, $p_1(p_y, w)$; $y = y[p_y, p_1(p_y, w), w]$ and $x_1 = x_1\{y[p_y, p_1(p_y, w), w], w\}$.

- $y = y[p_y, p_1(p_y, w), w]$; ex-post price of the sub-game $p_1\{y[p_y, p_1(p_y, w), w], w\}$ and $x_1 = x_1\{y[p_y, p_1(p_y, w), w], w\}$.

- or yet require y that insures agreement with the first-round price, $p_1(p_y, w) = p_1(y, w)$ generating $y = y(p_y, w)$ - then incompatible with supply $y[p_y, p_1(p_y, w), w]$ –, that can be imputed into $x_1\{y(p_y, w), p_1[y(p_y, w], w), w\}$, the effectively ordered quantity.

Of the three possibilities, the first is always better for 1 than the second.

4) Say, for a monopolist, (10) generates a contingent (sub-game) equilibrium contract, $x_1 = x_1[y, p_1(y, w), w] = x_1(y, w)$ placed by downstream producers that agree on price $p_1(y, w)$; such contract is going to be enforced. In the second bargaining round, y is discussed in connection with the production abilities of firm 1: firm 1 sets x_1, maximizing $p_1[y(p_y, x_1, w), w]\ x_1[y(p_y, x_1, w), w] – C^1(x_1, w)$, where $y(p_y, x_1, w)$ denotes supply of firm y; 1 generates a response obeying

$$\{x_1[y(p_y, x_1, w), w]\ \partial p_1[y(p_y, x_1, w), w]/\ \partial y + p_1[y(p_y, x_1, w), w]/\ \partial y\ \partial x_1[y(p_y, x_1, w), w]/\ \partial y\}\ \partial y(p_y, x_1, w)/\ \partial x_1 - \partial C^1(x_1,w)/\ \partial x_1 = 0$$

a solution $x_1 = x_1(p_y, w)$ that implies $y = y[p_y, x_1(p_y, w), w]$, inputed into the first-stage game: $p_1\{y[p_y, x_1(p_y, w), w], w\}$ and $x_1\{y[p_y, x_1(p_y, w), w], w\} = x_1\{y[p_y, x_1(p_y, w), w], p_1\{y[p_y, x_1(p_y, w), w], w\}$.

3.5. Upstream duopoly

For a duopoly, (10) establishes how i sets its price conditional on other variables, including the price of the other input: i's reaction function $p_i = p^i(p_j, y, w)$. On i's reaction function, for given y and w:

$$(\{2 - \partial^2 C^i[x_i(y, p_1, p_2, w), w]/\partial x_i^2\ \partial x_i(y, p_1, p_2, w)/\partial p_i\}\ \partial x_i(y, p_1, p_2, w)/\partial p_i + \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\ \partial^2 x_i(y, p_1, p_2, w)/\partial p_i^2\ )\ dp_i +$$

$$+ (\{1 - \partial^2 C^i[x_i(y, p_1, p_2, w), w]/\partial x_i^2\ \partial x_i(y, p_1, p_2, w)/\partial p_i\}\ \partial x_i(y, p_1, p_2, w)/\partial p_j + \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\ \partial^2 x_i(y, p_1, p_2, w)/\partial p_i \partial p_j\ )\ dp_j = 0$$

The term multiplying dp_i is negative for SOC to hold. For prices around marginal cost, the term multiplying dp_j has the sign of $\partial x_i(y, p_1, p_2, w)/\partial p_j$, positive if i and j are substitutes in production in the downstream technology - $p_i = p^i(p_j, y, w)$ is positively sloped in (p_1, p_2) space -, negative if they are complements - $p_i = p^i(p_j, y, w)$ is then negatively sloped.

With two duopolists, in equilibrium, $p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i$ reacts to p_i according to $1 - d\{\partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}/dp_i = 1 - \partial^2 C^i[x_i(y, p_1, p_2, w), w]/\partial x_i^2\ [\partial x_i(y, p_1, p_2, w)/\partial p_i + \partial x_i(y, p_1, p_2, w)/\partial p_j\ \partial p^j(p_i, y, w)/\partial p_i]$. $\partial x_i(y, p_1, p_2, w)/\partial p_j\ \partial p^j(p_i, y, w)/\partial p_i$ is always positive; if (but not only if) the own effect dominates in the sum in square brackets, we expect higher prices than in a price-taker equilibrium – yet lower than in a monopoly.

The geometry of reaction functions implicit in (10) – at given w and the other argument, say y – is well-known. For stability, i's reaction function should exhibit a higher (more negative) slope than j's in space (p_i, p_j) if both are positively (negatively) sloped – Fig. 3 (Fig. 4). Over i's reaction function, because its profits react to j's prices according to $\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\ \partial x_i(y, p_1, p_2, w)/\partial p_j$, i's profits rise (decrease) as p_j increases if i and j are substitutes (complements) – signalled in the Figs.

Also, the reaction curve of firm i, (10), at given p_j, y and w, will always imply a higher price than $p_i = \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i$ (because at the prices satisfying this, (10) is positive; as SOC determine that it decreases with p_i, this must raise to re-establish equality to 0): the latter will establish a pseudo-reaction function of a price-taker upstreamer – depicted in the Figs, $p_i = MC_i(p_j, y, w)$ - with slope $dp_i/dp_j = \{\partial^2 C^i[x_i(y, p_1, p_2, w), w]/\partial x_i^2\ \partial x_i(y, p_1, p_2, w)/\partial p_j\} / \{1 - \partial^2 C^i[x_i(y, p_1, p_2, w), w]/\partial x_i^2\ \partial x_i(y, p_1, p_2, w)/\partial p_i\}$ and that will lie to the left of i's reaction function in the (p_i, p_j)

space. It is immediate to conclude that the monopolistic equilibrium, E, will imply at least one of the prices larger than the price-taker environment, originating C.

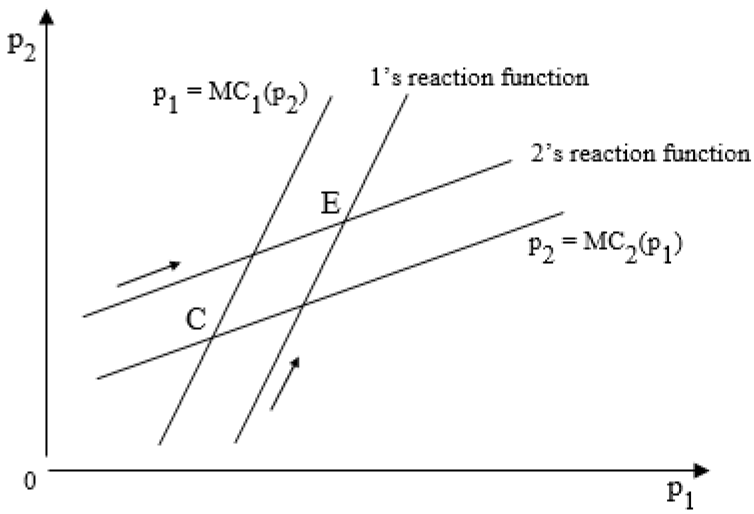

Figure 3. *1 and 2 Substitutes*

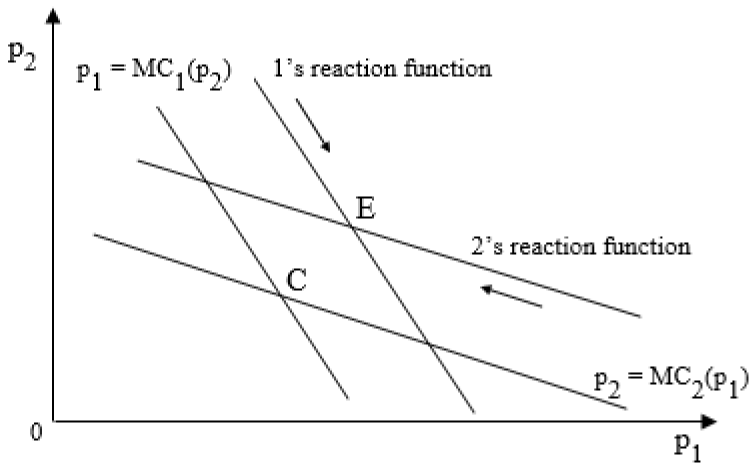

Figure 4. *1 and 2 Complements*

If we evaluate reaction curves at the optimal supply and depict $p_i = p^i[p_j, y(p_y, p_1, p_2, w), w]$, with slope $dp^i[p_j, y(p_y, p_1, p_2, w), w]/dp_j = [\partial p^i[p_j, y(p_y, p_1, p_2, w), w]/ \partial p_j + \partial p^i[p_j, y(p_y, p_1, p_2, w), w]/ \partial y\ \partial y(p_y, p_1, p_2, w)/ \partial p_j]/[1 - \partial p^i[p_j, y(p_y, p_1, p_2, w), w]/ \partial y\ \partial y(p_y, p_1, p_2, w)/ \partial p_i]$; signs of slopes may be preserved – given now by the numerator of the expression. Profits will increase (decrease) with p_j iff $\{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\}$ $\{\partial x_i(p_y, p_1, p_2, w)/ \partial p_j + \partial x_i(y, p_1, p_2, w)/ \partial y\ \partial y(p_y, p_1, p_2,$

$w)/\partial p_i\, dp^i[p_j, y(p_y, p_1, p_2, w), w]/dp_j\}$ – or $\partial x_i(y, p_1, p_2, w)/\partial p_j + \partial x_i(y, p_1, p_2, w)/\partial y\, \partial y(p_y, p_1, p_2, w)/\partial p_j + \partial x_i(y, p_1, p_2, w)/\partial y\, \partial y(p_y, p_1, p_2, w)/\partial p_i\, dp^i[p_j, y(p_y, p_1, p_2, w), w]/dp_j$ - is positive (negative); along the "new" i's reaction curve, i's profits rise (decrease) with p_j when 1 and 2 are substitutes (complements), once the sign of the expression is the same as that of $\partial x_i(p_y, p_1, p_2, w)/\partial p_j - \partial x_i(y, p_1, p_2, w)/\partial p_j\, \partial p^i[p_j, y(p_y, p_1, p_2, w), w]/\partial y + \partial x_i(y, p_1, p_2, w)/\partial y\, \partial p^i[p_j, y(p_y, p_1, p_2, w), w]/\partial p_i\}\, \partial y(p_y, p_1, p_2, w)/\partial p_i$.

As $y(p_y, p_1, p_2, w)$ answers to p_y – and input prices -, the further internalization of the downstream firm output scale optimization process also affects the equilibrium outcome. i's reaction function moves down in (p_i, p_j) space as quantity pre-commitment is lifted – i.e. as reaction functions change from (10) – evaluated at supply - to (14); therefore, we expect that a further internalization of scale optimization (at given w and p_j) decreases p_i (shifts towards the origin i's reaction function) and, therefore, will tend to increase x_i. Slopes of reaction functions are related to substitutability in derived demands and profit increases (decreases) with the other firm's price along a reaction function if they are substitutes (complements).

For duopolists, we can rely on reaction function geometry to infer that regardless of the commitment status of i, he will benefit from quantity commitment in the supply sense (over (10)) of j iff i and j are substitutes [12] – in Fig. 3, 2's reaction function is higher and therefore 1 reaches higher profits (on its reaction curve, held fixed) if 2 is pre-committed; it will benefit from its withdrawal if i and j are complements. Therefore, complementarity would point to the emergence of no pre-commitment of the invoked source, but with substitutability, pre-commitment with respect to y might improve the outcome of the bargaining exchange between the two duopolists.

The desire for such vertical restraints does not occur with upstream price-takers, i.e., that set $p_i = \partial C^i(x_i, w)/\partial x_i$ without taking into account any of the downstream firm feedback: if the downstream firm is competitive, its cost minimization will not be affected. They may arise again, however for the pre-fixing of the purchase of other inputs if some uncompetitive behavior towards them exist, though – argument which parallels the effects on short and long run cost curves…

[12] We are assuming that the sign of pre-commitment reaction function slopes are preserved with the replacement of supply...

Proposition 7:

If inputs are substitutes (complements) in downstream technology, quantity pre-commitment relative to a duopolist improves (worsens) a Bertrand competitor's position.

Perfect collusion will involve the setting of (p_1, p_2) maximizing the sum of profits of the two firms:

$$x_i(y, p_1, p_2, w) + \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\} \partial x_i(y, p_1, p_2, w)/ \partial p_i +$$
$$+ \{p_j - \partial C^j[x_j(y, p_1, p_2, w), w]/ \partial x_j\} \partial x_j(y, p_1, p_2, w)/ \partial p_i = 0 \qquad (27)$$

We may still look at (27) as implying pseudo-reaction functions of the two firms. Around the Bertrand equilibrium, over of the Bertrand competitor i's reaction function:

If i and j are substitutes, $\partial x_j(y, p_1, p_2, w)/ \partial p_i > 0$, expression (27) is positive: i's optimal pseudo reaction function will lie to the right – will imply a higher p_i for the same p_j - of the Bertrand's one.

If i and j are complements, $\partial x_j(y, p_1, p_2, w)/ \partial p_i < 0$, expression (27) is negative: i's optimal pseudo reaction function will lie to the left of Bertrand's one.

Obviously, conclusions apply at fixed y. In the absence of pre-commitment, analogous statements could be advanced for derived demands, conditional on p_y.

Proposition 8:

If inputs are substitutes (complements) in downstream technology, prices will be higher (lower) under upstream firm coordination than under Bertrand competition.

Additional knowledge of supply of final product would add to (27):

$$[\ \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/ \partial x_i\} \partial x_i(y, p_1, p_2, w)/ \partial y\ +$$
$$+ \{p_j - \partial C^j[x_j(y, p_1, p_2, w), w]/ \partial x_j\} \partial x_j(y, p_1, p_2, w)/ \partial y\ ]\ \partial y(p_y, p_1, p_2, w)/ \partial p_i \qquad (28)$$

Tying arrangements could be a device to achieve collusion. Some may as well be suggested with sequential optimization arguments similar to those suggesting our definition of quantity pre-commitment (Nash) bargaining. Let the downstream firm relevant demand be (a short-run demand) $x_i(y, p_i, x_j, w)$ - of course, ex-post (in the long-run) it will be evaluated at the (long-run) demand $x_j(y, p_i, p_j, w)$ [13] – or rather $x_i(p_y, p_i, p_j, w)$ and $x_j(p_y, p_i, p_j, w)$. Then, firm i will set $p_i = p^i(y, x_j, w)$ obeying:

$$x_i(y, p_i, x_j, w) + \{p_i - \partial C^i[x_i(y, p_i, x_j, w), w]/\partial x_i\}\, \partial x_i(y, p_i, x_j, w)/\partial p_i = 0 \quad (29)$$

evaluated at $x_j = x_j(y, p_1, p_2, w)$. (10) has an extra term relative to (29):

$$x_i(y, p_1, p_2, w) + \{p_i - \partial C^i[x_i(y, p_1, p_2, w), w]/\partial x_i\}\, \partial x_i(y, p_1, p_2, w)/\partial p_i = (30)$$

$$= (29) + \{p_i - \partial C^i[x_i(y, p_i, x_j, w), w]/\partial x_i\}\, \partial x_i(y, p_i, x_j, w)/\partial x_j\, \partial x_j(y, p_1, p_2, w)/\partial p_i$$

Evaluated at (29), (30) – hence (10) – is negative if $\partial x_i(y, p_i, x_j, w)/\partial x_j\, \partial x_j(y, p_1, p_2, w)/\partial p_i < 0$. If the two derivatives have opposite sign – guaranteeing that the long-run demand for x_i is more elastic than the short-run -, i's reaction curve (10) will be to the left of (29) in the (p_i, p_j) space. Then, equilibrium input prices will be higher if negotiations are tied – i.e., obey (29) rather than (10) - in the sense that decisions relative to one's price and quantity can be made conditional on the other's quantity, being settled jointly by y. Of course, the final outcome is not the collusion one, but the simultaneous bargaining of the two products – as a device for each upstream duopolist to negotiate on the, less elastic, short rather than long-term input demand - may improve profits – approach (27) - over the unilateral bargaining outcome (10) if they are substitutes…

Finally, integration of y with one of the upstream firms, say 1, would allow the solution (10) for i = 2 at $p_1 = \partial C^1[x_1(y, p_1, p_2, w), w]/\partial x_1$, that would suggest a new "reaction function" of 1, with a lower implicit price of input 1 than in any monopolistic competition arrangement at any given value of the other variables – $p_1 = MC_1(p_2)$ of Figs 2 and 3 -, generating equilibrium at its intersection with 2's reaction function. According to the slope of 2's reaction function, we therefore expect a lower price of an input with partial vertical integration with another if both of them are substitutes in the downstream technology [14] – a higher price if they are complements, in which case the independent intermediate product firm sees its profits enhanced.

[13] The firms are assumed to, nevertheless, set prices... In general, the resulting outcome will differ from the Cournot quantity-setting stage – explored in Singh & Vives (1984), for example. That would point to maximization in x_i by each firm of a profit function where price is replaced by the solution of the inverse system of the two long-run input demands.

[14] See Martin (1993), p. 247 and references in footnote 15.

Proposition 9:

If inputs are substitutes (complements), partial vertical integration will lower (raise) the price and profits of the other intermediate product. It always lower the shadow price of the integrated firm's product if long-run supply is more elastic than the short-run one.

4. Monopsonistic intermediate product buyers

Monopsonistic behavior of a downstream firm towards input producing firms would also be welfare detrimental and vertical integration necessarily welfare improving – and raise aggregate profits under perfectly elastic final demand and primary factor supplies. Firms will set:

$$p_y\, f_i(x_1,\ x_2,\ L_y)\ \partial x_i(p_i,\ w)/\partial p_i\ =\ x_i(p_i,\ w) + p_i\ \partial x_i(p_i,\ w)/\partial p_i \tag{31}$$

or

$$p_y\, f_i[x_1(p_1,\ w),\ x_2(p_2,\ w),\ L_y]\ =\ x_i(p_i,\ w)\ /\ \partial x_i(p_i,\ w)/\partial p_i\ +\ p_i$$

$$p_y\, f_L(x_1, x_2, L_y)\ =\ w \tag{32}$$

$$p_i\, g^1(L_i)'\ =\ w \quad \text{from where (or)} \quad x_i\ =\ x_i(p_i, w) \tag{33}$$

Replacing (32) in (31) would allow to derive pseudo-reaction functions suggesting how concavity – and substitutability - in the downstream technology affects the inputs price choice. Given the passive position of upstream firms – already price-takers -, "competitive interaction" between them is not a good description of the scenario. Welfare qualification therefore proceeds for a single intermediate input and $y = f(x_1, L_y)$. Then, from (31), we immediately conclude that, as $\partial x_i(p_i, w)/\ \partial p_i > 0$, the value of marginal product of input 1 will tend to be higher and, therefore, the use of input 1 lower and – because $\partial x_i(p_i, w)/\ \partial p_i > 0$ - the intermediate input price lower under the monopsonist arrangement.

The introduction of another downstream seller facing the upstream price-taker allows the analysis of duopsony behavior towards an homogeneous intermediate product. Now $x_1^A + x_1^B\ =\ x_1(p_1, w)$, where A and B refer the downstream firms and x_1^l firm l's purchases of x_1. We can write its inverse as $p_1 = p_1(x_1^A + x_1^B, w)$; most likely, quantity competition will be observed. Then, FOC require (33) and

$$p_y\, f_1(x_1^l, L_y^l)\ =\ p_1(x_1^A + x_1^B, w) + x_1^l\, \partial p_1(x_1^A + x_1^B, w)/\ \partial x_1\ ,\ l = A, B \tag{34}$$

If L_y is fixed, (34) establishes l's reaction function, l = A, B; on it:

$$[p_y f_{11} - 2\,\partial p_1(x_1^A + x_1^B, w)/\,\partial x_1 - x_1^l\,\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1^2]\, dx_1^l =$$
$$= [\partial p_1(x_1^A + x_1^B, w)/\,\partial x_1 + x_1^l\,\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1^2]\, dx_1^k,\ l = A, B \tag{35}$$

The left hand-side is negative – by SOC; provided $\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1^2$ is positive or not too negative, reaction functions will be negatively sloped.

If p_y rises, at a given x_1^k, l's reaction function shifts outwards [15]: a rise in the output price will imply an increase in intermediate input purchases – hence a higher intermediate input price.

Also, if L_1 is normal in the upstream technology, $\partial p_1(x_1^A + x_1^B, w)/\,\partial w = - [\partial x_1(p_1, w)/\,\partial w] / [\partial x_1(p_1, w)/\,\partial p_1] > 0$; then, provided $\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1 \partial w$ is positive or not too negative, reaction curves shift inwards after a rise in wages implying lower quantities – hence a lower intermediate input price [16].

If L_y^l is allowed to adjust, the left hand-side of (35) becomes:

$$[p_y f_{11} - p_y f_{1L}^2/ f_{LL} - 2\,\partial p_1(x_1^A + x_1^B, w)/\,\partial x_1 - x_1^l\,\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1^2]\, dx_1^l \tag{36}$$

The effect of a change in p_y would have the same sign. For w, we would consider:

$$[p_y f_{11} - p_y f_{1L}^2/ f_{LL} - 2\,\partial p_1(x_1^A + x_1^B, w)/\,\partial x_1 - x_1^l\,\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1^2]\, dx_1^l = [\partial p_1(x_1^A + x_1^B, w)/\,\partial w + x_1^l\,\partial^2 p_1(x_1^A + x_1^B, w)/\,\partial x_1 \partial w - f_{1L}/f_{LL})]\, dw \tag{37}$$

As $f_{LL} < 0$, if (but not only if) $f_{1L} > 0$, the sign of the wage effect remains.

[15] (Some) normality in the use of input 1 by the downstream technology would eventually be required if L_y was adjustable.

[16] Note that we have only two inputs, and they must be substitutes in production. With more inputs, the result would be observed under complementarity between L_y and x_1 in downstream technology – with substitutability allowing the reverse to occur.

Proposition 10:

Monopsonistic downstream, quantity-setter firms towards an intermediate product price-taker:

10.1. will set intermediate prices below the competitive level.

10.2. will increase intermediate product prices after a rise in output price; (under reasonable assumptions) will decrease them after a rise in wages.

The problem could also be stated in terms of conditional demands.

It is clear that relative to the downstream price-taker conditional demands $x_i(y, p_1, p_2, w)$ and $L_y(y, p_1, p_2, w)$, the monopsonist will achieve now their values with the replacement of p_i by $x_i(p_i, w) / \partial x_i(p_i, w)/ \partial p_i + p_i$:

$$x_i[y, p_1 + x_1(p_1, w) / \partial x_1(p_1, w)/ \partial p_1, p_2 + x_2(p_2, w) / \partial x_2(p_2, w)/ \partial p_2, w],\ i = 1,2$$

$$L_y[y, p_1 + x_1(p_1, w) / \partial x_1(p_1, w)/ \partial p_1, p_2 + x_2(p_2, w) / \partial x_2(p_2, w)/ \partial p_2, w] \tag{38}$$

that – as they are stated in implicit form - determine equilibrium with the requirement that supplies of intermediate products are equated to demands. For simplicity, assume a single intermediate input. Then:

$$x_1[y, p_1 + x_1(p_1, w) / \partial x_1(p_1, w)/ \partial p_1, w] = x_1(p_1, w) \tag{39}$$

allows for

$$p_1 = p_1(y, w) \tag{40}$$

Given that demand responds negatively to the second argument, at the price-taker solution for p_1, p_1^C – for which $x_1(y, p_1^C, w) = x_1(p_1^C, w)$ - $x_1[y, p_1^C + x_1(p_1^C, w) / \partial x_1(p_1^C, w)/ \partial p_1, w]$ - $x_1(p_1^C, w) < 0$. For the wedge to close, as it responds to p_1 according to $\partial x_1(y, p_1, w)/ \partial p_1$ $\{2 - x_1(p_1, w)\ \partial^2 x_1(p_1, w)/ \partial p_1^2\ /[\partial x_1(p_1, w)/ \partial p_1]^2\}$ - $\partial x_1(p_1, w)/ \partial p_1$ - negative if $\partial^2 x_1(p_1, w)/ \partial p_1^2$ is negative or not too positive - the monopsonist p_1 will most likely be below the competitive level.

The competitive cost function $C(y, p_1, p_2, w)$ is replaced by an implicit form $p_1\, x_1(p_1, w) + p_2\, x_2(p_2, w) + w\, L_y[y, p_1 + x_1(p_1, w) / \partial x_1(p_1, w)/ \partial p_1, p_2 + x_2(p_2, w) / \partial x_2(p_2, w)/ \partial p_2, w]$ or $p_1\, x_1[y, p_1 + x_1(p_1, w) / \partial x_1(p_1, w)/ \partial p_1, p_2 + x_2(p_2, w) / \partial x_2(p_2, w)/ \partial p_2, w] + p_2\, x_2[y, p_1 + x_1(p_1, w) /$

$\partial_1(p_1, w)/\partial p_1, p_2+x_2(p_2, w) / \partial x_2(p_2, w)/\partial p_2, w] + w\, L_y[y, p_1+x_1(p_1, w) / \partial x_1(p_1, w)/\partial p_1, p_2+x_2(p_2, w) / \partial x_2(p_2, w)/\partial p_2, w]$.

Final output quantity restraints would allow - on (40) - for $p_1 = p_1\{y[p_y, p_1+ x_1(p_1, w) / \partial x_1(p_1, w)/\partial p_1, w], w\}$ and would be *ex-post* irrelevant – because the decision-maker that faces reaction is the same and internalizes all the feedback that effectively opposes him: from the structure of FOC of the profit maximization problem we conclude that he would choose demand y according to the competitive supply, $y(p_y, p_1, w)$, but evaluated (in implicit form) at $[p_y, p_1+ x_1(p_1, w) / \partial x_1(p_1, w)/\partial p_1, w]$, which he recognizes to set x_1.

Oppositely, one can easily show that in the presence of multiple arguments of the upstream production function, the downstream monopolist will benefit from short-run flexibility of the upstream firm relative to all primary factors quantities - even if rigidity would hinder the elasticity of the input supply effectively faced by the downstream monopolist in negotiations. Let the intermediate input 1 technology use two factors L_a and L_b with only the first adjustable in the short-run (ignore L_y in y's production); the short-run supply is $x_1(p_1, w_a, L_b)$, while the long run is $x_1(p_1, w_a, w_b) = x_1[p_1, w_a, L_b(p_1, w_a, w_b)]$ where $L_b(p_1, w_a, w_b)$ is the (long-run) derived demand for input L_b. If bargaining is set-up in the short-run while L_b is pre-settled, FOC of the downstream firm imply:

$$[p_y f_1(x_1) - p_1]\, \partial x_1(p_1, w_a, L_b)/\partial p_1 - x_1(p_1, w_a, L_b) = 0 \tag{41}$$

If L_b can be adjusted in the short-run, the condition is replaced by:

$$\begin{aligned}&[p_y f_1(x_1) - p_1]\, \partial x_1(p_1, w_a, L_b)/\partial p_1 - x_1(p_1, w_a, L_b) + \\ &+ [p_y f_1(x_1) - p_1]\, \partial x_1(p_1, w_a, L_b)/\partial L_b\, \partial L_b(p_1, w_a, w_b)/\partial p_1 = 0\end{aligned} \tag{42}$$

At the short-run solution, $p_y\, f_1(x_1) > p_1$. Then, as $\partial x_1(p_1, w_a, L_b)/\partial L_b\, \partial L_b(p_1, w_a, w_b)/\partial p_1$ is expected to be positive – if short run supply is less elastic than the long-run one -, the left hand-side of (42) is positive at the solution satisfying (41): p_1 will be set at higher level - implying a higher $x_1(p_1, w_a, w_b)$ – than if only the short-run response was present (even if evaluated at $L_b(p_1, w_a, w_b)$).

The impact on y's profits of the additional flexibility is negative. We are comparing situations where, ex-post, L_b is fixed at level $L_b(p_1, w_a, w_b)$; allowing for upstream short-run adjustment feedbacks negatively into y's rent appropriation ability because the value of marginal product of input 1 is set higher than its price. But, because p_1 rises, also the upstream market profits rise.

Proposition 11:

Monopsonistic downstream, quantity-setter firms towards an intermediate product price-taker:

11.1. will find quantity restraints irrelevant.

11.2. will benefit and induce an increase of intermediate product prices with more flexible hiring of upstream production.

For duopsonists quantity setters – that do not have to produce the same final output -, with generic competitive demands $x_1^l = x_1^l(y_l, p_1, w)$, l = A,B, conditional reaction functions will obey

$$x_1^l = x_1^l[y_l, p_1(x_1^A + x_1^B, w) + x_1^l\, \partial p_1(x_1^A + x_1^B, w)/\partial x_1, w] \tag{43}$$

$$\{1 - \partial x_1^l[y_l, p_1(x_1^A + x_1^B, w) + x_1^l\, \partial p_1(x_1^A + x_1^B, w)/\partial x_1, w]/\partial p_1 \\ [2\, \partial p_1(x_1^A + x_1^B, w)/\partial x_1 + x_1^l\, \partial^2 p_1(x_1^A + x_1^B, w)/\partial x_1^2]\}\, dx_1^l = \\ = \partial x_1^l[y_l, p_1(x_1^A + x_1^B, w) + x_1^l\, \partial p_1(x_1^A + x_1^B, w)/\partial x_1, w]/\partial p_1 \\ [\partial p_1(x_1^A + x_1^B, w)/\partial x_1 + x_1^l\, \partial^2 p_1(x_1^A + x_1^B, w)/\partial x_1^2]\, dx_1^k,\ l = A, B \tag{44}$$

They will be negatively sloped – see Fig. 5 - provided $\partial^2 p_1(x_1^A + x_1^B, w)$/◎◎$x_1^2$ is positive or not too negative. The competitive pseudo-reaction curve of a competitive firm l, $x_1^l = x_1^l[y_l, p_1(x_1^A + x_1^B, w), w]$ – function $x_1^l = X_l(x_1^k)$ in Fig. 5 -, will imply, at the same x_1^l, a higher x_1^k: on (43), $x_1^l = x_1^l[y_l, p_1(x_1^A + x_1^B, w) + D, w]$, where D is positive; fixing x_1^l on this function, to change D, x_1^k has to react according to $\partial D / \partial x_1^k = - \partial p_1(x_1^A + x_1^B, w)/ \partial x_1 < 0$; then to decrease D to 0, x_1^k has to increase: at the same x_1^l, x_1^k of the competitive buyer must be higher.

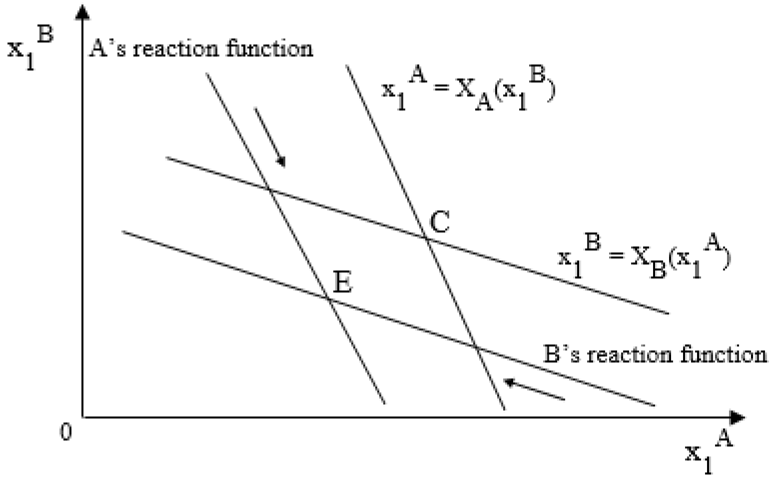

Figure 5. *Duopsony*

If y rises and 1 is normal (regressive) in downstream technology, at x_1^k, l's reaction function shifts outwards (inwards): a rise in the output will imply an increase (decrease) in intermediate input purchases – hence a higher (lower) intermediate input price.

Also, if L_1 is normal in the upstream technology, as $\partial p_1(x_1^A + x_1^B, w)/\partial w = -\partial x_1(p_1, w)/\partial w / \partial x_1(p_1, w)/\partial p_1$ evaluated at $p_1(x_1^A + x_1^B, w)$, $\partial p_1(x_1^A + x_1^B, w)/\partial w > 0$; then, provided $\partial^2 p_1(x_1^A + x_1^B, w)/\partial x_1 \partial w$ is not too negative, being x_1 and L_y complements or not too substitutes in downstream technology, reaction curves shift inwards after a rise in wages implying lower quantities – hence a lower intermediate input price (regressivity of L_1 and substitutability between x_1 and L_y would lead to the reverse).

Partial integration with one of the firms – leading to the intersection of its competitive equation with the other's reaction function - will raise the downstream merged firm production and decrease the other's quantity and profits.

Proposition 12:

Monopsonistic downstream, quantity-setter firms towards an intermediate product price-taker:

12.1. generate Cournot-type reaction function behaviour in the intermediate input purchases.

12.2. partial vertical integration will raise the downstream merged firm production and decrease the other's quantity and profits.

5. Monopolistic primary factor market (owners)

Assume that the factor market is non-competitive and only input 1 enters the downstream firm's production function – in which, other arguments – factors - than x_1 and L_1 may exist. Demands and supplies are also subject to prices of these background inputs, assumed parametrically fixed throughout the section.

Unions – factor owners - seek to maximize wage bills, w_1 L_1 and or w_y L_y. They may or may not set different wages across the two firms (plants, in case of integration).

As in the previous section towards input producers, we note now that the unions' decision concerning wage is constrained by the perceived demand they face – or by the antecedent contractual arrangements with regard to other firms' decision variables.

Case 1: Competitive Input Market

Under competitive unintegrated markets, unions may set different wages in the two firms. If $L_y(y, p_1, w_y)$ is the conditional demand of the downstream firm, union y will seek to maximize w_y $L_y(y, p_1, w_y)$, setting w_y such that

$$L_y(y, p_1, w_y) + w_y \, \partial L_y(y, p_1, w_y)/ \partial w_y = 0 \tag{45}$$

implying

$$w_y = - L_y(y, p_1, w_y) / \partial L_y(y, p_1, w_y)/ \partial w_y \quad \text{or} \quad w_y = w_y(y, p_1)$$

From (45), a wage-setting function $w_y = w_y(y, p_1)$ is derived. Being L_y normal in the downstream technology, we expect that $\partial w_y(y, p_1)/ \partial y = -[\partial L_y(y, p_1, w_y)/ \partial y + w_y \, \partial^2 L_y(y, p_1, w_y)/(\partial y \partial w_y)] / [2 \, \partial L_y(y, p_1, w_y)/ \partial w_y + w_y \, \partial^2 L_y(y, p_1, w_y)/ \partial w_y^2] > 0$ – which will occur if $\partial^2 L_y(y, p_1, w_y)/(\partial y \partial w_y)$ w_y has a smaller impact than or favorable to that of $\partial L_y(y, p_1, w_y)/ \partial y$ in the numerator; and – under the same condition with respect to p_1 - being L_y and x_1 substitutes (complements [17]), $\partial w_y(y, p_1)/ \partial p_1 = -[\partial L_y(y, p_1, w_y)/ \partial p_1 + w_y \, \partial^2 L_y(y, p_1, w_y)/(\partial p_1 \partial w_y)] / [2 \, \partial L_y(y, p_1, w_y)/ \partial w_y + w_y \, \partial^2 L_y(y, p_1, w_y)/ \partial w_y^2] > (<) 0$.

[17] With only two inputs, they will necessarily be substitutes; with additional inputs, the statement stands.

The upstream firm, with labor demand $L_1(p_1, w_1)$ - arising from $p_1 g^1(L_1)' = w_1$ if firm 1 only uses input L_1 -, will set:

$L_1(p_1, w_1) + w_1\, \partial L_1(p_1, w_1)/\partial w_1 = 0$, implying $w_1 = w_1(p_1)$

(46) establishes w_1 as a function of p_1 or vice-versa. Being L_1 normal in the upstream technology, we expect $w_1(p_1)' = -\, [\partial L_1(p_1, w_1)/\partial p_1 + w_1\, \partial^2 L_1(p_1, w_1)/(\partial w_1 \partial p_1)] / [2\, \partial L_1(p_1, w_1)/\partial w_1 + w_1\, \partial^2 L_1(p_1, w_1)/\partial w_1^2] > 0$ – guaranteed if the second derivative of demand in the numerator multiplied by w_1 has a smaller impact than or favorable to that of first one.

Given that the firms behave competitively – and that wages may be set independently -, (45), (46) and equilibrium between supply and demand for input 1:

$$x_1(p_1, w_1) = g^1[L_1(p_1, w_1)] = x_1(y, p_1, w_y) \tag{47}$$

define equilibrium at given y. In fact, (47) implies a relation

$$p_1 = p_1(y, w_1, w_y) \tag{48}$$

It is immediate to show that $\partial p_1(y, w_1, w_y)/\partial A$ has the sign of $\partial x_1(y, p_1, w_y)/\partial A$ for $A = y, w_y$: positive (negative) for $A = y$ if 1 is a normal (regressive) input in the downstream technology; positive (negative) for $A = w_y$ if 1 and L_y are substitutes (complements) in the downstream technology. $\partial p_1(y, w_1, w_y)/\partial w_1$ - with the sign of $-\,\partial x_1(p_1, w_1)/\partial w_1$ – will be positive (negative) if L_1 is normal in the upstream technology.

(48) can be inserted in (46), generating from $w_1 = w_1[p_1(y, w_1, w_y)]$ an implicit or observable reaction function of union 1 to union y's wage, $w_1 = w_1(y, w_y)$. Again, (provided L_1 is normal and so $w_1(p_1)' > 0$) if x_1 and L_y are substitutes (complements) in the downstream technology, it will be positively (negatively) sloped: $\partial w_1(y, w_y)/\partial w_y > (<) 0$. If x_1 is normal (regressive) in the downstream technology (again, provided L_1 is normal in upstream production), $\partial w_1(y, w_y)/\partial y > (<) 0$.

Likewise, (48) can be inserted in (45), generating from $w_y = w_y[y, p_1(y, w_1, w_y)]$ an implicit reaction function of union y to union 1's wage, $w_y = w_y(y, w_1)$. Slopes are more difficult to derive. Provided $1 - \partial w_y(y, p_1)/\partial p_1\, \partial p_1(y, w_1, w_y)/\partial w_y > 0$ (and L_1 is normal in upstream technology), if x_1 and L_y are substitutes (complements) in the downstream technology, it will be positively (negatively) sloped. Under the same condition, $\partial w_y(y, w_1)/\partial y > 0$ when both x_1 and L_y are normal if but not only if x_1 and L_y are substitutes.

(1 - $\partial w_y(y, p_1)/\partial p_1\ \partial p_1(y, w_1, w_y)/\partial w_y > 0$, iff $[2\ \partial L_y(y, p_1, w_y)/\partial w_y + w_y\ \partial^2 L_y(y, p_1, w_y)/\partial w_y^2]\ [\partial x_1(y, p_1, w_y)/\partial p_1 - [\partial x_1(p_1, w_1)/\partial p_1] > [\partial x_1(y, p_1, w_y)/\partial w_y + w_y\ \partial^2 x_1(y, p_1, w_y)/(\partial w_y \partial p_1)]\ \partial x_1(y, p_1, w_y)/\partial w_y$. Concavity of the cost function $C(y, p_1, w_y)$ in (p_1, w_y) would suggest its likelihood.)

The two cases are then depicted in Figs. 6 and 7. The wage bill of each union rises according to the arrows direction: y's wage bill changes with w_1 along y's reaction curve according to $w_y\ \partial L_y(y, p_1, w_y)/\partial p_1\ [\partial p_1(y, w_1, w_y)/\partial\ \partial w_1 + \partial p_1(y, w_1, w_y)/\partial w_y\ \partial w_y(y, w_1)/\partial w_1]$ – under normality, signed as $\partial L_y(y, p_1, w_y)/\partial p_1$; 1's wage bill changes with w_y along 1's reaction curve according to $w_1\ \partial L_1(y, p_1)/\partial p_1\ [\partial p_1(y, w_1, w_y)/\partial w_y + \partial p_1(y, w_1, w_y)/\partial w_1\ \partial w_1(y, w_y)/\partial w_y]$ – under normality, signed as $\partial w_1(y, w_y)/\partial w_y$.

E is the equilibrium point.

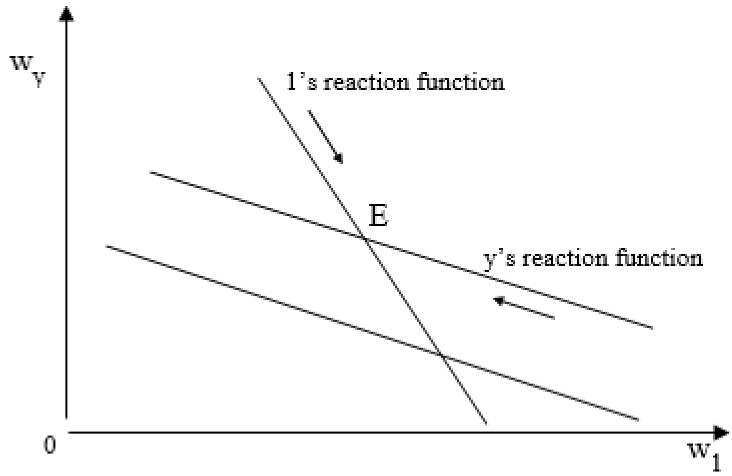

Figure 6. *1 and L_y Substitutes*

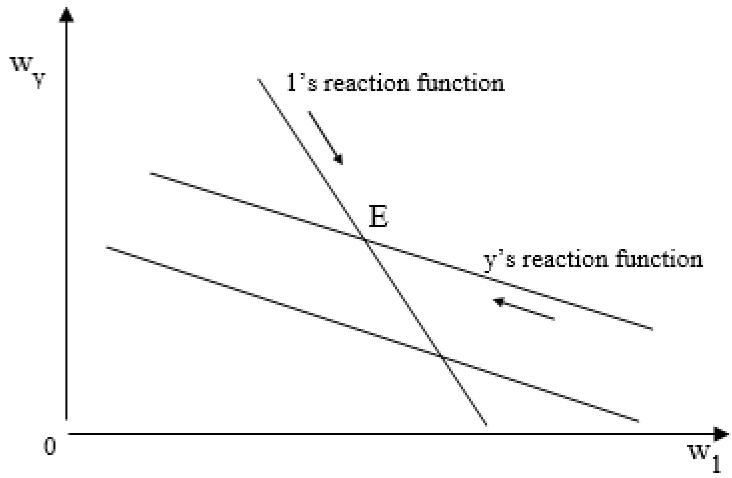

Figure 7. *1 and L_y Complements*

Also, provided that $L_y[y, p_1(y, w_1, w_y), w_y]$ reacts negatively to w_y – or labor market stability based on an excess demand adjustment mechanism is guaranteed -, for an interior solution (45) to be possible, it must imply lower employment than the (a) positively sloped labor supply does at the same wage: at a given w_1, the monopoly wage w_y must be higher than the competitive one, i.e., $w_y(w_1)$ that would insure equality between supply and demand; such "competitive" function $w_y(w_1)$ would be below y's reaction curve in space (w_1, w_y) in Figs 6 and 7. Analogously, w_1 obeying (46) – at given w_y – should be higher than the competitive level.

Proposition 13:

In vertically decentralized markets where firms behave competitively, monopolistic – revenue-maximizers - primary factor owners will

13.1. likely optimize conditional on the ex-post equilibrium intermediate input price.

13.2. implicitly compete setting prices (wages) above the competitive level, reacting to each other through positively (negatively) sloped wage reaction functions if the intermediate input and the factor are substitutes (complements) in the downstream technology.

13.3. increase wages after a rise in final output if the intermediate input and the factor are substitutes in the downstream technology. The upstream union will raise wages but the downstream one may raise or contract them after a rise in final output in case of complementarity.

In the absence of final output quantity pre-commitment, (45) is replaced by:

$L_y(y, p_1, w_y) + w_y\, \partial L_y(y, p_1, w_y)/\partial w_y + w_y\, \partial L_y(y, p_1, w_y)/\partial y\, \partial y(p_y, p_1, w_y)/\partial w_y = 0$, implying, replacing y by $y(p_y, p_1, w_y)$, $w_y = w_y(p_y, p_1)$

or

$$L_y(p_y, p_1, w_y) + w_y \, \partial L_y(p_y, p_1, w_y)/\partial w_y = 0 \quad (49)$$

where $y(p_y, p_1, w_y)$ denotes the downstream firm supply. $\partial L_y(y, p_1, w_y)/\partial y$ and $\partial y(p_y, p_1, w_y)/\partial w_y$ will have opposite sign. Hence, at the solution satisfying (45) and $y(p_y, p_1, w_y)$, condition (49) is negative – at a fixed p_1 (and given p_y), union y will choose a lower wage w_y relative to the case of pre-commitment, (45).

(47) is replaced by

$$x_1(p_1, w_1) = g^1[L_1(p_1, w_1)] = x_1[y(p_y, p_1, w_y), p_1, w_y] = x_1(p_y, p_1, w_y) \quad (50)$$

where $x_1(p_y, p_1, w_y)$ is the derived demand for x_1, and therefore, (48) by:

$$p_1 = p_1[y(p_y, p_1, w_y), w_1, w_y]\text{, generating } p_1 = p_1(p_y, w_1, w_y) \quad (51)$$

to be replaced in (49) and in $w_1 = w_1(p_1)$, from (46), which remains valid.

If we replaced (50) and the supply function – or (51) - in $w_1 = w_1(p_1)$ and in $w_y = w_y(y, p_1)$ from (45), we would obtain the implicit "final" reaction functions of the pre-committed case. Provided that $x_1(p_y, p_1, w_y)$ reacts to p_y, w_1 and w_y in the same direction as before $x_1(y, p_1, w_y)$ responded to y, w_1 and w_y, the implicit $p_1 = p_1[y(p_y, p_1, w_y), w_1, w_y] = p_1(p_y, w_1, w_y)$ reacts to p_y, w_1 and w_y as before $p_1(y, w_1, w_y)$ did to y, w_1 and w_y (Because $1 - \partial p_1(y, w_1, w_y)/\partial y \; \partial y(p_y, p_1, w_y)/\partial p_1 > 0$, it is immediate to verify that such is necessarily the case for p_y – supply is positively sloped – and w_1); then, reaction function geometry would still be valid – with substitutability and complementarity qualified in terms of derived demands.

Now, relative to the reaction functions with pre-commitment, y's reaction function derived from (49) and (50) implies a lower w_y - in Figs. 6 and 7, y's reaction function would be below the one of the pre-committed case in the (w_1, w_y) space. Then, both factor prices will be lower without pre-commitment if L_y and 1 are substitutes; the upstream wage will be higher and the downstream one lower if they are complements. Union 1 will be better (worse) off with pre-commitment if L_y and 1 are substitutes (complements) in the downstream technology; union y may end-up worse off without it in any case.

Proposition 14:

In vertically decentralized markets where firms behave competitively, and primary factor owners are monopolistic revenue-maximizers, factor prices will be higher in the presence of final output commitment if the factor and the input are substitutes in the downstream technology. The upstream wage will be lower and the downstream wage higher in case of complementarity.

Case 2: Vertically Integrated Markets

With an integrated industry, on the one hand, firms may not be able to justify different wages – in which case, the equilibrium will surely differ from the previous one. On the other, there is no intermediate disguise of competition between unions in wage setting, no competition layer provided by the intermediate product market. And different strategic behavior will imply different equilibrium: now unions set wages conditional on either wages or quantities – or other – of the other one, and not *at a given level of the value of marginal product of the intermediate product in firm 1* as was hypothesized for decentralized markets.

Unions may behave as Bertrand competitors. Then,

$$L_y(y, w_1, w_y) + w_y\, \partial L_y(y, w_1, w_y)/\partial w_y = 0 \tag{52}$$

$$L_1(y, w_1, w_y) + w_1\, \partial L_1(y, w_1, w_y)/\partial w_1 = 0 \tag{53}$$

Well the first condition will differ from (45) evaluated at $p_1(y, w_1, w_y)$, coming from the equilibrium condition (48):

$$L_y[y, p_1(y, w_1, w_y), w_y] + w_y\, \partial L_y[y, p_1(y, w_1, w_y), w_y]/\partial w_y = 0 \tag{54}$$

where $L_y[y, p_1(y, w_1, w_y), w_y]$ represents $L_y(y, p_1, w_y)$ evaluated at $p_1(y, w_1, w_y)$. Rather, $L_y(y, w_1, w_y) = L_y[y, p_1(y, w_1, w_y), w_y]$ and (52) will correspond to:

$$\begin{aligned} &L_y[y, p_1(y, w_1, w_y), w_y] + w_y\, \partial L_y[y, p_1(y, w_1, w_y), w_y]/\partial w_y + \\ &\quad + w_y\, \partial L_y[y, p_1(y, w_1, w_y), w_y]/\partial p_1\, \partial p_1(y, w_1, w_y)/\partial w_y = 0 \end{aligned} \tag{55}$$

Then, $\partial L_y[y, p_1(y, w_1, w_y), w_y]/\partial p_1\, \partial p_1(y, w_1, w_y)/\partial w_y$ is always positive, with the two terms having the same sign: positive (negative) if x_1 and L_y are substitutes (complements). At the same level w_1 (and y), the optimal w_y should be higher than that of (45) – union y's reaction curve is above the one of the decentralized case in space (w_1, w_y).

Also, (53) will differ from (46), including relative to the latter the term $w_1\, \partial L_1[p_1(y, w_1, w_y), w_1]/ \partial p_1\, \partial p_1(y, w_1, w_y)/ \partial w_1$, always positive. That implies that at the same level w_y, the optimal w_1 should be higher than over (46) – union 1's reaction curve is, in space (w_1, w_y), to the right of the one of the decentralized case.

Then, both w_1 and w_y are expected to be higher in the integrated industry.

Proposition 15:

In vertically integrated markets where firms behave competitively, and monopolistic primary factor owners are revenue-maximizers:

15.1. their strategic behavior may vary substantially and from the vertically decentralized industry.

15.2. if they are Bertrand competitors, they implicitly compete setting prices (wages) above the competitive level, reacting to each other through positively (negatively) sloped wage reaction functions if the intermediate input and the factor are substitutes (complements) in the downstream technology. Both factor prices are higher than in a decentralized industry

Or they may collude. In this case, wages will be set in a way such that:

$$L_y(y, w_1, w_y) + w_y\, \partial L_y(y, w_1, w_y)/ \partial w_y + w_1\, \partial L_1(y, w_1, w_y)/ \partial w_y = 0 \quad (56)$$
$$L_1(y, w_1, w_y) + w_1\, \partial L_1(y, w_1, w_y)/ \partial w_1 + w_y\, \partial L_y(y, w_1, w_y)/ \partial w_1 = 0 \quad (57)$$

Being the two inputs x_1 and L_y substitutes (complements), the last terms are positive (negative) and suggest a further wage increase (decrease) of both factor prices relative to the Bertrand case – known effects in the literature. Then, in case of substitutability, vertical integration with Bertrand competition, by raising wages, will promote an increase in wage bills relative to the decentralized solution.

In the latter case, they will probably be constrained to set the same wage for the different tasks:

$$L_y(y, w_1, w_y) + w_y\, \partial L_y(y, w_1, w_y)/ \partial w_y + w_1\, \partial L_1(y, w_1, w_y)/ \partial w_y +$$
$$+ L_1(y, w_1, w_y) + w_1\, \partial L_1(y, w_1, w_y)/ \partial w_1 + w_y\, \partial L_y(y, w_1, w_y)/ \partial w_1 = 0 \quad (58)$$

at $w_1 = w_y = w$. At such wage, (56) and (57) are symmetric: if (56) is positive, the optimal w_y is larger than w_1 in the unrestricted case and vice-versa if it is negative.

Identical analysis could be preformed for the derived demands – for absence of quantity restraints. Conclusions would not alter qualitatively under input normality – substitutability qualified in terms of derived demands.

6. Monopsonistic primary factor market

Of course, the complementary view to the previous market structure, of monopsonistic behaviour toward supply of labor could generate an opposite assessment of decentralization, but only if supplies are not independent. If supplies of factor L to the upstream and downstream firms are unrelated - $L^y(w_y)$ and $L^1(w_1)$ -, they work as independent factors – represent different primary inputs that may be priced differently – and provided firms behave competitively with respect to all other decisions, industry structure is irrelevant. It will not be, if wages are constrained to equalize – say, union bargaining requires (nothing else than…) wage equalization. Then, instead of

$$p_y\, f_1(x_1, L_y) = p_1 \tag{59}$$

$$p_y\, f_L(x_1, L_y) = L^y(w_y) / L^y(w_y)' + w_y \tag{60}$$

$$p_1\, g^1(L_1)' = L^1(w_1) / L^1(w_1)' + w_1 = p_y\, f_1(x_1, L_y)\, g^1(L_1)' \tag{61}$$

with final solutions given by (60) and (61), a vertically integrated industry observes:

$$p_y\, f_1(x_1, L_y)\, g^1(L_1)'\, L^1(w)' + p_y\, f_L(x_1, L_y)\, L^y(w)' = \\ = L^1(w) + L^y(w) + w\, [L^1(w)' + L^y(w)'] \tag{62}$$

Proposition 16: In vertical markets where firms behave competitively in intermediate product markets but are monopsonists towards primary factor markets:

16.1. equilibrium is independent of vertical arrangements provided that factor prices (supplies) are unrelated.

16.2. if factor prices must equalize under vertical integration, profits will decrease with the merger in the industry.

Suppose instead that it is the same factor that provides both firms, with inverse aggregate supply $W(L_1 + L_y)$. Then, a decentralized market will achieve:

$$p_y\, f_1(x_1, L_y) = p_1 \tag{63}$$

$$p_y\, f_L(x_1, L_y) = W(L_1 + L_y) + W(L_1 + L_y)'\, L_y \tag{64}$$

$$p_1 \, g^1(L_1)' = W(L_1 + L_y) + W(L_1 + L_y)' \, L_1 \tag{65}$$

(65) establishes a reaction function of firm 1, $L_1 = L^1(p_1, L_y)$; it can be transformed in a pseudo-reaction function conditional on p_y by solving for L_1 the equation $L_1 = L^1\{p_y \, f_1[g^1(L_1), L_y], L_y\}$, obeying

$$p_y \, f_1[g^1(L_1), L_y] \, g^1(L_1)' = W(L_1 + L_y) + W(L_1 + L_y)' \, L_1 \tag{66}$$

The implicit $L_1 = L^1(p_y, L_y)$ will probably be negatively sloped provided that $W(L_1 + L_y)' + W(L_1 + L_y)'' \, L_1 > p_y \, f_{1L}[g^1(L_1), L_y] \, g^1(L_1)'$. (63) and (64) – replacing x_1 from one of them – suggest a reaction function of the downstream firm at given p_y; alternatively, one can replace $g^1(L_1)$ on (64) and derivate $L_y = L^y(p_y, L_1)$:

$$p_y \, f_L[g^1(L_1), L_y] = W(L_1 + L_y) + W(L_1 + L_y)' \, L_y \tag{67}$$

as the downstream firm counteracting reaction function; it will be negatively sloped provided that $W(L_1 + L_y)' + W(L_1 + L_y)'' \, L_y > p_y \, f_{L1}[g^1(L_1), L_y] \, g^1(L_1)'$, i.e., under a similar condition to that under which 1's is.

A vertically integrated market would achieve a different equilibrium, requiring:

$$p_y \, f_L[g^1(L_1), L_y] = W(L_1 + L_y) + W(L_1 + L_y)' \, (L_1 + L_y) = \tag{68}$$

$$p_y \, f_1[g^1(L_1), L_y] \, g^1(L_1)' = W(L_1 + L_y) + W(L_1 + L_y)' \, (L_1 + L_y) \tag{69}$$

(68) and (69) can still be seen to suggest quantity competition between the purchase of the two inputs. On (68), because $W(L_1 + L_y)' > 0$, at a given level of L_1, L_y must be to the left of the level at which was on (67). Likewise, on (69): at a given level of L_y, L_1 must be to the left at the level at which was on (66). So – regardless of the slopes of the reaction functions -, employment is expected to be lower than in the decentralized economy.

Being $W(L_1 + L_y)' > 0$, wages are expected to be lower for a vertical integrated industry.

Proposition 17:

In vertical markets where two firms behave competitively in intermediate product markets, but are monopsonists towards the primary factor market that supplies homogeneous labor:

17.1. a decentralized equilibrium will allow for competition for factor quantities by the two firms.

17.2. vertical integration will imply lower employment and wages than decentralized settings.

7. Uncertainty in intermediate product industries

In this section we illustrate the impact of exogenous uncertainty affecting directly the intermediate product unit on the market outcome. To the extent that it affects the equilibrium solution, we justify vertical integration on such grounds. We consider that the intermediate product market is competitive – price is set at marginal cost – and maintain that firms maximize profits – are risk-neutral - to isolate pure uncertainty effects.

Uncertainty is introduced as an exogenous Bernoulli lottery, Z, of null expected value: with probability q it takes the value s, with probability (1 – q) it takes the value $-\frac{qs}{1-q}$, so that E[Z] = 0 and Var(Z) = $\frac{qs^2}{1-q}$. Three alternative sources of uncertainty are considered to affect the intermediate product market: factor quantity uncertainty – and the exogenous randomness affects additively the production function –, factor quality uncertainty – Z is added to the argument of the intermediate product firm production function [18] –, and wage uncertainty.

A first note to be made is that if there is ex-post flexibility to the realization of Z – price of input 1 is formed after s or $-\frac{qs}{1-q}$ is observed -, equilibrium – and aggregate profits which then rise with uncertainty in prices - is invariant to vertical arrangements. Of course, neither the equilibrium quantities [19], nor the "split" of industry profits between the two in the case of decentralized markets will be invariant to uncertainty.

Ex-ante commitment with respect to quantity decisions may imply different choices in a vertically integrated and in a decentralized market. It will not with wage or final output price uncertainty, i.e., if Z is added to wages or prices: in either industry arrangement, market outcome is invariant to such uncertainty. But it will in other cases:

Factor quantity uncertainty, i.e., $x_1 = g^1(L_1) + Z$ does not affect demand for $L_1(p_1, w_1)$ in a decentralized market equilibrium; $p_1\ g^1(L_1)' = w$ and

[18] See Feldstein (1971) for a similar concept of technological uncertainty, yet working multiplicatively.

[19] See Martins (2005) for single input technology case.

firm 1's supply is $x_1(p_1, w_1) + Z$[20]. Being prices fixed ex-ante, firm y takes quantity $x_1(p_1, w_1) + Z$; she maximizes

$$p_y\, E[f(x_1 + Z, L_y)] - p_1\, x_1 - w_y\, L_y =$$
$$= p_y\, [q\, f(x_1 + s, L_y) + (1-q)\, f(x_1 - \frac{qs}{1-q}, L_y)] - p_1\, x_1 - w_y\, L_y \tag{70}$$

so that

$$p_y\, [q\, f_1(x_1 + s, L_y) + (1-q)\, f_1(x_1 - \frac{qs}{1-q}, L_y)] = p_1$$
$$p_y\, [q\, f_L(x_1 + s, L_y) + (1-q)\, f_L(x_1 - \frac{qs}{1-q}, L_y)] = w_y$$
$$p_1\, g^1(L_1)' = w_1$$

and therefore the equilibrium obeys:

$$p_y\, \{q\, f_1[g^1(L_1) + s, L_y] + (1-q)\, f_1[g^1(L_1) - \frac{qs}{1-q}, L_y]\} = p_1 \tag{71}$$
$$p_y\, \{q\, f_L[g^1(L_1) + s, L_y] + (1-q)\, f_L[g^1(L_1) - \frac{qs}{1-q}, L_y]\} = w_y \tag{72}$$
$$p_1\, g^1(L_1)' = w_1 \tag{73}$$

An integrated downstream firm would set L_1 and L_y such that it would maximize:

$$p_y\, E\{f[g^1(L_1) + Z, L_y]\} - w_1\, L_1 - w_y\, L_y \tag{74}$$

setting:

$$p_y\, \{q\, f_1[g^1(L_1) + s, L_y] + (1-q)\, f_1[g^1(L_1) - \frac{qs}{1-q}, L_y]\}\, g^1(L_1)' = w_1 \tag{75}$$
$$p_y\, \{q\, f_L[g^1(L_1) + s, L_y] + (1-q)\, f_L[g^1(L_1) - \frac{qs}{1-q}, L_y]\} = w_y \tag{76}$$

Obviously, the market solution is the same as for the decentralized market. Equilibrium would equivalently apply if the uncertainty was felt

[20] That different firms may exist in the market is compatible with a measure Z independent of their number, just requiring that it represents perfectly and positively correlated uncertainty across all the firms enduring it.

directly and additively to x_1 in firm y production function and prices were pre-committed.

Factor quality uncertainty, i.e., $x_1 = g^1(L_1 + Z)$ would imply

$$p_y\ \{q\, f_1[g^1(L_1 + s), L_y] + (1 - q)\, f_1[g^1(L_1 - \frac{qs}{1-q}), L_y]\} = p_1 \tag{77}$$

$$p_y\ \{q\, f_L[g^1(L_1 + s), L_y] + (1 - q)\, f_L[g^1(L_1 - \frac{qs}{1-q}), L_y]\} = w_y \tag{78}$$

$$p_1\ [q\, g^1(L_1 + s)' + (1 - q)\, g^1(L_1 - \frac{qs}{1-q})'] = w_1 \tag{79}$$

(77) and (79) imply:

$$p_y\ \{q\, f_1[g^1(L_1 + s), L_y] + (1 - q)\, f_1[g^1(L_1 - \frac{qs}{1-q}), L_y]\}$$
$$[q\, g^1(L_1 + s)' + (1 - q)\, g^1(L_1 - \frac{qs}{1-q})'] = w_1 \tag{80}$$

A vertically integrated market will set

$$p_y\{q\, f_1[g^1(L_1+s),L_y]g^1(L_1+s)'+(1-q)\, f_1[g^1(L_1-\frac{qs}{1-q}),L_y]g^1(L_1-\frac{qs}{1-q})'\} = w_1 \tag{81}$$

$$p_y\ \{q\, f_L[g^1(L_1 + s), L_y] + (1 - q)\, f_L[g^1(L_1 - \frac{qs}{1-q}), L_y]\} = w_y \tag{82}$$

As $g^1(L_1)$ is concave and $f_{11} < 0$ for f to be concave, one can show that the left hand-side of (80) is always smaller than that of (81): at the decentralized industry solution, the derivative (of the vertical integrated firm's problem) generating (81) is positive and the integrated market will tend to exhibit a higher L_1.

Aggregate expected profits are certainly lower under decentralization – under market integration, a certain degree of flexibility is gained in internal pricing of the intermediate input: with vertical merger the intermediate price stickiness is circumvented. Of course, for the market itself not to allow price fluctuations, input quality must only be realized after internal use in the downstream production process.

Proposition 18:

Assuming up and downstream firms are risk-neutral profit maximizers and behave competitively:

18.1. Under ex-post flexibility, aggregate market outcomes are invariant to vertical arrangements, even if not to uncertainty.

18.2. With ex-ante commitment, wage uncertainty has no effect on market outcomes.

18.3. Ex-ante quantity uncertainty in the production of the downstream firm, affects market outcomes. The impact is invariant to vertical arrangements.

18.4. Ex-ante quality uncertainty in the production of the downstream firm, affects market outcomes and differently according to industry vertical structure. Aggregate profits are lower under vertical decentralization; the vertically integrated market will produce more intermediate product – and potentially choose a larger size.

8. Summary and conclusions

Analysis of industry vertical structure usually neglects primary factor effects. This study identified some of the consequences of specific market competition arrangements when explicit and complete production functions of both downstream and upstream producers are accounted for.

A first interesting consequence is the mathematical characterization of technical economies of depth – defined with respect to technology features of the existing downstream technology only. The following analysis was always performed assuming their absence and mostly a perfectly elastic final demand. It suggested stylized unilaterally uncooperative arrangements likely to affect the equilibrium of a decentralized vertical industry:

Monopolistic upstream markets will set input prices above marginal cost. Intermediate input prices will lower with downstream scale flexibility, introduced as derived input demand rather than conditional demand-based optimization – (both) downstream (and upstream) firms will benefit from the removal of such vertical restraints. Partial integration will lower the price of a substitute upstream duopolist; it will increase that of a complement (which will benefit with integration). With a less than perfectly elastic final demand its feedback would alter some of the conclusions; then, resale price maintenance clauses would appear to promote a downstream monopoly/cartel competition and thus enhance derived intermediate input demand faced by upstream firms, which may then benefit from its imposition.

Vertical restraints were modelled allowing downstream autonomy towards final output market. Some comments applied when that hypothesis can no longer be assumed valid – suggesting the generation of delegation type equilibria.

A monopsonist intermediate product buyer will set input prices below the competitive level. Vertical restraints on final supply would be redundant; conversely, short-run flexibility in upstream hiring decisions will benefit a downstream monopsonist – and the upstream price-taker.

Monopolistic primary factor owners set their prices above their competitive level. The downstream wage will be higher in the presence of vertical (final output) restraints - further output contraction responds to the downstream union wage increases without them; the upstream wage will be higher (lower) if the intermediate input is substitute (complement) to the primary factor in the downstream technology. With integration - and wage-competition between unions -, both wages will likely rise.

Vertical integration would not affect monopsonistic primary factor markets if factor supplies are unrelated and factor prices still allowed to differ – i.e., unless monopsonistic price discrimination is no longer possible. If wages must always equalize, vertical integration will lower wages – and total employment: the competition layer provided by the decentralized exchange of intermediate product is lost and production of final output contracts.

Of final relevance, a formal justification for vertical integration in the presence of uncertainty towards an intermediate product was produced, based on its quality uncertainty – and even if its exogenous source remains active in the two scenarios -, with vertical integration suggesting a higher production level. Quantity uncertainty or ex-post price flexibility would render vertical arrangements irrelevant.

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