Journal of Economics Library
www.kspjournals.org
Volume 5 December 2018 Issue 4

The future of economic growth in the world's largest economies

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Abstract. The future of economic growth is projected by solving differential equations describing growth rate. Analysis was carried out for 12 countries representing the leading economies responsible for around 70% of the global economic output. Out of all these countries, the most secure and stable economic growth is in Japan, Germany and France. In contrast, economic growth in China, India and Brazil is strongly insecure and potentially leading to the economic collapse. Economic growth in the United States, United Kingdom, Canada and Australia is on the border line. It also might become unsustainable. Economic growth in the remaining two countries, Italy and Russian Federation, is unpredictable. As for the preventive measures, for Japan, Germany and France, growth rate should be, if possible, maintained at a small value below 1%. Economic growth in these countries is described by logistic trajectories. Their asymptotic approach to a maximum value is hard to control but the growth rate should not be allowed to be substantially increased. For China, India and Brazil, growth rate should be now decreasing sufficiently fast to avoid the potential economic collapse. For the USA, UK, Canada and Australia, it would be also advisable to decrease their growth rate faster than in the recent years. For two countries, Italy and Russian Federation, it is essential to stabilise, if possible, their economic growth.
Keywords. Gross Domestic Product; Future Economic Growth; Sustainable Economic Growth; Economic Collapse; USA, China, Japan, Germany, France, UK, Brazil, India, Italy, Canada, Russian Federation, Australia.
JEL. A10, B41, C02, C20, C50, F00, F01.

1. Introduction

The aim of this study is to investigate the future of economic growth. Countries contributing most to the global economic growth are listed in Table 1. This Table is based on the World Bank data (World Bank, 2017). The total of the Gross Domestic Product (GDP) for these leading economies was \$52.029 trillion in 2010 US\$, which represented 69.5% of the global GDP.

Mathematical method of this analysis is based on solving the following type of the differential equations:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = F\,, \tag{1}$$

where $S(t)$ is the size of the GDP and F is the mathematical description of the empirical growth rate, which could be a function of time or the function of $S(t)$.

It is also useful to introduce the economic stress factor, which can be defined by the following expression:

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$$\sigma \equiv \frac{GDP(t)}{GDP(t=2000)}. \quad (2)$$

It is simply the ratio of the GDP at any given time t to the GDP in the year 2000. The economic stress factor $\sigma = 2$ means that whatever was produced and consumed in one year in 2000 will have to be produced and consumed in half a year. The stress factor $\sigma = 12$ means that production/consumption of one year in 2000 would have to be compressed to one month to support a given trajectory of growth, while $\sigma = 365$ would mean the production/consumption of one year in the year 2000 would have to be compressed to one day at a given time.

Obviously, there is a limit to growth even with unlimited natural resources because there is a limit to how much can be produced and consumed during a given section of time. Considering these limitations, the probability of reaching a higher level of economic growth, as expressed by the GDP, might be lower in countries where the GDP/cap (Gross Domestic Product per capita) is already high than in countries where it is low but it does not mean that it will be easier to reach a higher level of the GDP in poorer countries because there are also other limitations such as limitations imposed by the availability of natural resources or the limitations in the production efficiency.

Table 1. *The leading economies included in this study*

Economy	*GDP 2015 [Trillion 2010 US$]*	*GDP/cap [2010 US$]*
USA	16.597	51,638
China	8.909	6,497
Japan	5.986	47,150
Germany	3.697	45,408
France	2.777	41,534
UK	2.683	41,188
Brazil	2.317	11,159
India	2.295	1,751
Italy	2.058	33,489
Canada	1.793	50,000
Russian Federation	1.616	11,159
Australia	1.301	54,708
Total	52.029	
Global Total	74.889	
Fraction of Global	69.5%	

Source: World Bank (2017); GDP/cap: GDP per capita

The general tendency or desire everywhere is to increase growth rate as much as possible but this is now a serious mistake. Even constant growth rate should be watched closely because it generates exponential growth, which depending on the level of the growth rate, can increase rapidly and become unsustainable.

It is essential to understand that the decreasing growth rate should not be interpreted as the decreasing economic growth. As long as the growth rate is positive, the size of the GDP will continue to increase even if the growth rate is decreasing. The GDP will decrease only if the growth rate is negative and it will continue to decrease only if the growth rate remains negative. Fluctuations between positive and negative values of the growth rate will only produce the GDP approximately constant.

2. Examples of the application of this mathematical method

2.1. Growth of the world population

According to the data presented by the US Census Bureau (2018), growth rate for the growth of the global human population has been gradually decreasing from around 1963. Analysis of the growth rate suggests that it might be decreasing asymptotically to zero. Based on this analysis, the most likely growth trajectory is described by the pseudo-logistic growth given by the following equation:

$$S(t) = C\exp\left[\frac{a}{b}e^{bt}\right], \tag{3}$$

with parameters $a = 2.179\times10^{10}$ and $b = -1.406\times10^{-2}$. This trajectory approaches asymptotically the normalisation constant C, which in this case is 15.6 billion. The projected population in 2200 is 14.7 billion, only around 1 billion below its asymptotic value. This projected growth is based on using the full range of growth rate data and is most reliable. If a less likely linear approximation is used for the growth rate from the year 2000, then the generated trajectory is given by the second-order exponential growth:

$$S(t) = \exp(a_0 + a_1 t + a_2 t^2), \tag{4}$$

with parameters $a_0 = 1.222\times10^{-1}$, $a_1 = 2.520\times10^{-1}$ and $a_2 = -5.585\times10^{-5}$. This projected distribution reaches a maximum of 11.9 billion in 2105. Both projections are shown in Figure 1. (All Figures are in the Appendix.)

Calculations shown in Figure 1 are in excellent agreement with projections published by United Nations (2015). According to this source "The world population is projected to increase by more than one billion people within the next 15 years, reaching 8.5 billion in 2030, and to increase further to 9.7 billion in 2050 and 11.2 billion by 2100" (United Nations, 2015, p. 2). Predictions shown in Figure 3 are 8.4 billion in 2030, 9.8 billion in 2050 and 11.8 billion in 2100 for the trajectory leading to the localised maximum. If the growth of the world population is going to follow the trajectory leading to the asymptotic maximum, then it will reach 8.4 billion in 2030, 9.8 billion in 2050 and 12.4 billion in 2100. The two predicted trajectories are identical over a long time and not until the end of the current century or even the beginning of the next century might we be able to know whether the growth of the world population is going to reach a localised maximum and start to decrease or whether it will continue to increase towards its larger asymptotic maximum. All these calculations are, of course, based on the assumption that the growth of population can be supported.

Summary of all these predictions is presented in Table 2. The UN projection gives no information about the expected size of human population in the 22nd century. For the 21st century, the agreement between these two independent predictions is remarkably good.

However, there is also a possibility that the growth rate will not be decreasing asymptotically to zero but to a constant positive value. Such a situation is, for instance, in the growth of population in China. In this case, the growth of the world population will never reach a maximum (asymptotic or localised) but will continue to increase exponentially. Such a growth would be definitely unsustainable.

The best option, if there is an option, would be to try to slow down the growth of the world population even more than now experienced. However, it is hardly

expected that such a global undertaking will be ever attempted, or even if undertaken that it would be successful. It is hard to control the growth of a large size of population, and an excellent example is China. They have made a determined effort to control the growth of their population and they managed to reduce their growth rate to around 0.5% from a maximum of 1.6% in 1988 (World Bank, 2017). Their growth rate remained constant at around 0.5% for the past 10 years, but recently it started showing signs of a gradual increase.

Table 2. *Predicted growth of the world population*

Source	*2030*	*2050*	*2100*	S_{max}	S_a
UN	8.5	9.7	11.2	NI	NI
CA	8.4	9.8	11.8	11.9	NA
CA	8.4	9.8	12.4	NA	15.6

Notes: UN – United Nations, (2015); CA – current analysis (Nielsen 2017b); NI – no information; NA – not applicable; S_{max} – maximum value; S_a – asymptotic value

The current population in China is around 1.4 billion, the largest population of a single country. This enormous size was reached after countless years of growth and despite of the recent drastic efforts to slow it down. If the growth of population in China is going to continue at the constant rate of only 0.5% per year, as it did in the past 10 years, the same size of the population, 1.4 billion, will be added in just 140 years, and then, after the next 140 years, the size of the population in China would double from 2.8 billion of 5.6 billion, all this with just the growth rate of only 0.5% per year.

The power and the danger of the exponential growth is generally not appreciated. Small annual percentage of growth might sound safe but it is not safe. The danger of the exponential growth is repeatedly overlooked, particularly in the economic growth where the general aim is to increase the economic growth rate or at least to keep it high. High economic growth rate is greeted with jubilation but it should be taken as a warning sign of a potentially unsustainable growth.

2.2. Preventable economic collapse in Greece

Economic collapse in Greece around 2008 could have been prevented and there were two clear warning signs. First it was when the growth rate was decreasing too fast. The second warning sign was when after reaching a low minimum the growth rate was increasing too fast.

The decreasing growth poses no danger to the economic growth but it should not be decreasing too fast. There is a certain optimal way for the growth rate to decrease, the way, which can be best determined by solving differential equation (1). If the growth rate is decreasing too fast it will lead to a low level of the GDP. If it is decreasing too slowly, it might lead to a dangerously fast and unsustainable economic growth.

In Greece, economic growth rate was decreasing too fast, from around 10% in 1961 to around 1% in 1990. Economic growth was following a logistic trajectory, which was approaching asymptotically a constant value. However, because the growth rate was decreasing too fast, there was not enough time to reach a sufficiently high level of the GDP. The GDP increased from around $45 billion, expressed in the 2005 US currency (World Bank, 2015) to only around $160 billion in 1990.

And now came the second stage of the dangerous economic growth: the growth rate started to increase too fast. This new pattern was generating a pseudo-hyperbolic trajectory escaping to infinity at a fixed time. Such a trajectory, which can be easily generated by the increasing growth rate, should be if possible avoided because it can quickly lead to a runaway process. The time of the escape to infinity

in Greece was in 2017. It was obviously impossible to follow such a trajectory for two long and, not surprisingly, the economic growth in Greece collapsed in around 2008.

Mathematical analysis of the growth rate could have been helpful in avoiding this dramatic and undesirable outcome. Now, it is essential to learn from the past experience to make sure that the same mistake is not repeated.

Economic growth in Greece is shown in Figure 2. The logistic trajectory is given by the following equation:

$$S(t)=\left[Ce^{-a_0 t}-\frac{a_1}{a_0}\right]^{-1}, \tag{5}$$

with parameters $C=3.002\times10^{130}$, $a_0=1.553\times10^{-1}$ and $a_1=-9.112\times10^{-4}$. For this empirically-determined set of parameters, the asymptotic limit to growth is around \$170 billion (of 2005 US\$).

The pseudo-hyperbolic trajectory is described by the same equation but with the positive value for a_1. Parameters describing the trajectory shown in Figure 2 are: $C=-4.215\times10^{-59}$, $a_0=-6.424\times10^{-2}$ and $a_1=4.839\times10^{-4}$. The singularity is given by the following expression:

$$t_s=-\frac{1}{a_0}\ln\frac{a_1}{a_0 C}. \tag{6}$$

The best recommended option for Greece now is to increase the economic growth rate, if possible, but slowly to avoid the earlier runaway process. When the GDP reaches acceptable level, economic growth rate should then be slowly, if possible, reduced. Any attempt to increase the growth rate too fast could lead again to the runaway process, as experienced earlier, and to a new economic collapse.

3. Projecting economic growth

3.1. United States of America

Economic growth in the USA is shown in Figure 3. Economic growth rate was steadily decreasing, generating the second-order exponential growth [see eqn (4)], leading to a maximum of \$25.9 trillion (of 2010 US\$) in 2060, which would be about twice as high as the GDP in 2000. However, considering that the GDP/cap in the US is already high, doubling the economic output might be difficult and consequently it is advisable to decrease the economic growth rate faster than in the past. Parameters describing this second-order exponential growth are: $a_0=-9.158\times10^{2}$, $a_1=8.924\times10^{-1}$ and $a_2=-2.167\times10^{-4}$.

3.2. China

Economic growth in China is shown in Figure 4. It was approximately exponential with the rate of around 9.5% per year, which corresponds to the doubling time of only 7.3 years Exponential growth is described by the following equation:

$$S(t)=Ce^{rt}, \tag{7}$$

where C is the normalisation constant related to the constant of integration and r is the growth rate. Parameters describing exponential growth in China are: $C = 6.700 \times 10^{-83}$ and $r = 9.500 \times 10^{-2}$.

Closer examination of the recent trend of the growth rate suggests that from around 1980, it might have started to decrease. If this part of the growth rate data is used to calculate economic growth trajectory, it generates the second-order exponential growth shown in Figures 5 and 6. Its parameters are: $a_0 = -1.641 \times 10^{3}$, $a_1 = 1.546 \times 10^{0}$ and $a_2 = -3.626 \times 10^{-4}$. This new trajectory leads to a maximum of around \$1,277 trillion (of 2010 US\$), which corresponds to the economic stress factor $\sigma = 570$. Figure 6 shows that even this slower growth is too fast because it dwarfs the earlier fast exponential growth.

China has every right to try to increase their GDP/cap but the question is whether higher level of the GDP/cap is achievable by following the past pattern of the fast growth. For richer countries listed in Table 1, the average GDP/cap is \$45,639. With the current population of China of around 1.4 billion, to reach the same level of economic status, China's GDP would have to be around \$64 trillion rather than around 9 trillion as recorded for 2015. Such a high GDP would correspond to the economic stress factor $\sigma = 29$, which might be tolerable providing that there is enough time to adjust to such a high-intensity economic output. The fast-increasing economic growth in China makes this goal difficult and probably even impossible to achieve. This level of economic stress would be reached in China in around 2035, if it follows the past exponential growth, or in around 2040 if it follows the slower second-order exponential trajectory. In this short time of around 20 or 25 years, counting from 2015, the economic output of one year in 2015 would have to be generated every 15 days.

Willingly or unwillingly, economic growth in China is likely to slow down. The best option for this country is to make this slowing-down process controllable by starting to reduce their growth sufficiently fast. Economic growth will be increasing but at least, with some care, it might follow a safely increasing trajectory.

3.3. Japan

Japan represents the most secure and the most sustainable economic growth in this group of countries. Their growth rate has been steadily decreasing and its pattern generates the logistic growth of the GDP, which is shown in Figure 7. It should be noted that the curve reproducing data was not calculated by fitting logistic distribution to the GDP values but independently by the analysis of the growth rate. This example can be, therefore, seen as a test of the introduced here method of mathematical analysis and of predicting growth trajectories. Parameters describing this logistic trajectory are: $C = 3.946 \times 10^{71}$, $a_0 = 8.411 \times 10^{-2}$ and $a_1 = -1.279 \times 10^{-2}$.

The logistic maximum is \$6.573 (2010 US\$), which is only 10% higher than in 2015 and only 23% higher than in 2000. This is a very secure and sustainable economic growth.

The best option for Japan is, if possible, to keep the economic growth rate close to zero. Japan should not follow the unfortunate example of other countries, which try to increase their economic growth rate. If the growth rate is increased in Japan, it should be closely monitored and regulated. It should not be increased for too long. On no account it should be steadily increasing. With the positive growth rate maintained close to zero, Japan will safely approach and maintain the predicted asymptotic maximum of the GDP and will enjoy a long economic security. Any

prolonged constant value of growth rate, even if small, should be avoided. Logistic growth is difficult to maintain, because ideally growth rate should be decreasing gradually to zero. It is, therefore, essential to regulate carefully, if possible, the growth rate to maintain such a growth.

3.4. Germany

Economic growth in Germany, shown in Figure 8, follows also a secure logistic trajectory. Its parameters are: $C = 4.979 \times 10^{35}$, $a_0 = 4.211 \times 10^{-2}$ and $a_1 = -8.429 \times 10^{-3}$ Its asymptotic value is \$4,995 trillion (2010 US\$), which is 60% higher than the GDP in 2000 ($\sigma = 1.60$). Economic growth rate in 2015 was approaching 1% per year. This is already close to the asymptotic value of zero.

The recommended option for Germany is to continue decreasing slowly the growth rate, if possible, with the aim of coming close to the asymptotic value of the GDP. Like Japan, Germany is now at the stage when the growth rate should be maintained close to zero and any attempt to increase it substantially should be avoided.

3.5. France

Economic growth in France, shown in Figure 9, is in the similar stage as in Germany and Japan, but Japan is in a more advanced stage of approaching the asymptotic logistic maximum. Parameters describing logistic growth in France are: $C = 6.027 \times 10^{44}$, $a_0 = 5.255 \times 10^{-2}$ and $a_1 = -1.570 \times 10^{-2}$. The projected logistic maximum is \$3.347 trillion (2010 US\$), which is 43% higher than the GDP in 2000 ($\sigma = 1.43$). France is also now at the stage when the growth rate should be close to zero and the economic growth should be carefully steered to reach its asymptotic value.

3.6. United Kingdom

Economic growth in the United Kingdom is shown in Figure 10. It also follows a logistic trajectory but unlike the growth in Japan, France and Germany, it is still far from the stage when the growth rate is close to 1%. Parameters describing logistic growth in the UK are: $C = 3.380 \times 10^{29}$, $a_0 = 3.461 \times 10^{-2}$ and $a_1 = -6.404 \times 10^{-3}$. The asymptotic maximum is \$5.405 trillion (2010 US\$), which corresponds to $\sigma = 2.60$. This value represents a significantly higher economic stress level than in Japan, Germany and France. Considering that economic output per person is already high in the UK, such a substantial increase in the economic stress might be hard to tolerate and maintain. A safer option for the UK would be to try to reduce, if possible, their economic growth rate faster than in recent years.

3.7. Brazil

From around 1985, growth rate describing economic growth in Brazil was steadily increasing, which might have been seen as a desirable feature leading to the increasing economic strength of this country. However, the opposite is true because such a growth is definitely unsustainable. If continued, it will inevitably lead to the economic collapse.

There are two possible growth trajectories generated by this increasing growth rate: (1) the ever-increasing and unsustainable, second-order exponential trajectory and (2) the unsustainable pseudo-hyperbolic trajectory escaping to infinity in 2053. They are shown in Figure 11. If the economic growth in Brazil is going to follow the pseudo-hyperbolic trajectory, economic collapse will happen before 2053. The danger is similar to the danger in Greece before their economic collapse around 2008. If the economic growth in Brazil is going to follow the second-order exponential growth, economic collapse might be only delayed but not avoided.

Parameters describing the second-order exponential trajectory are: $a_0 = 6.858\times10^{2}$, $a_1 = -7.152\times10^{-1}$ and $a_2 = 1.863\times10^{-4}$. Parameters describing the pseudo-hyperbolic growth are: $C = 3.088\times10^{15}$, $a_0 = 1.781\times10^{-2}$ and $a_1 = 7.296\times10^{-3}$.

Recently, as indicated by the GDP data, Brazil experienced a small economic decline. This could be just a temporary feature but it could be a sign of unsustainable growth even at this stage. If the economic growth is going to continue as before, the GDP will continue to increase to an inevitable economic collapse. The best and safe option for Brazil is to start decreasing, if possible, their economic growth rate but to keep it positive. The increasing growth rate might be seen as a sign of a strong economic growth but it is, in fact, a sign of the impeding economic collapse. Such an increase can be, at best, tolerated only for a short time.

3.8. India

Economic growth in India is even more insecure than in Brazil. Here also growth rate was steadily increasing, which might have been interpreted as a progress to prosperity but it is a progress to an assured calamity. Economic growth in India is shown in Figure 12.

As in Brazil, the increasing growth rate generates two possible trajectories: (1) the ever-increasing second order exponential trajectory or (2) the pseudo-hyperbolic trajectory. However, unlike the economic growth in Brazil, the projected pseudo-hyperbolic trajectory escapes to infinity much earlier, in 2029. If the growth rate in India is going to continue to increase, as it did in the past, India is progressing quickly to serious economic crisis.

India has a strong need to increase their GDP because their GDP/cap is exceptionally low (see Table 1). However, the road to the increased GDP should not be supported by the increasing or even high economic growth rate because such an approach is likely to lead to the economic collapse. In order to have a secure economic future, India should now start to decrease their growth rate but to keep it positive. The road to prosperity might be longer but safer.

Parameters describing the second order exponential growth in India are: $a_0 = 1.811\times10^{3}$, $a_1 = -1.875\times10^{0}$ and $a_2 = 4.848\times10^{-4}$. Parameters describing the pseudo-hyperbolic trajectory are: $C = 1.893\times10^{33}$, $a_0 = 3.801\times10^{-2}$ and $a_1 = 2.264\times10^{-2}$.

3.9. Italy

Economic growth in Italy, shown in Figure 13, was following a steadily increasing pseudo-logistic trajectory [see eqn (3)]. Its asymptotic maximum of $3 trillion (2010 US$) corresponded to $\sigma = 1.41$. It was a safe growth, but then, unexpectedly and unpredictably, it collapsed around 2008. New forces were introduced to the economic growth and the previously steadily-increasing growth was violently terminated. Under these conditions, future economic growth is unpredictable. It is an excellent example of how a perfectly safe, secure and stable economic growth can be changed into a disaster.

Parameters describing the pseudo-logistic growth pattern are: $C = 3.040\times10^{0}$, $a = 6.526\times10^{29}$ and $b = -3.644\times10^{-2}$.

3.10. Canada

Canada follows an unsustainable pattern of growth. From 1960, growth rate describing economic growth in Canada was decreasing hyperbolically. Such a growth rate generates the following growth trajectory:

$$S(t) = C(a + bt)^{1/b}. \quad (8)$$

Parameters describing economic growth in Canada, shown in Figure 14, are: $C = 3.476 \times 10^{-3}$, $a = -1.222 \times 10^{3}$ and $b = 6.321 \times 10^{-1}$. This is an unsustainable trajectory because it is ever increasing. To support this growth, σ would have to increase to around 2.3 in 2050 and to around 4 in 2100. Such a substantial increase might be hard to achieve because Canada's output per person in already high (see Table 1). Furthermore, there is no maximum for this trajectory. It is unrealistic to expect that economic growth in Canada will be ever increasing. A change will have to have to be made, at a certain stage and it would be better if the change is suitably controlled to avoid a possible sudden interruption in the growth trajectory. It would be advisable for Canada to start to decrease their growth rate suitably faster than in the past few years.

3.13. Russian Federation

Economic growth in Russian Federation is shown in Figure 15. The data are only from 1989. Economic growth in Russian Federation is strongly unstable and unpredictable.

3.14. Australia

Economic growth in Australia is shown in Figure 16. Even though it is described by a decelerating second-order exponential trajectory leading to a maximum, it is increasing too fast to be securely sustainable. Its maximum of \$4.672 trillion (2010 US\$) in 2111 corresponds to $\sigma = 5.52$, which should be compared with $\sigma = 1.52$ in 2015. Thus, in 2015, economic stress factor was only 52% higher than in 2000, but by 2111 it would have to be 452% higher to support this fast growth. As for the USA, UK and Canada, economic output in 2015 was already high and it might be difficult to increase it so much higher. The future of the economic growth in Australia is uncertain and to make it secure, growth rate should be now decreasing faster than in the recent past. Parameters describing the economic growth trajectory in Australia are: $a_0 = -6.100 \times 10^{2}$, $a_1 = 5.793 \times 10^{-1}$ and $a_2 = -1.372 \times 10^{-4}$.

4. Summary, discussion and conclusions

Results of mathematical analysis of economic growth in the leading economies listed in Table 1 are summarised in Table 3. Italy and Russian Federation are not included because their economic growth is unpredictable.

This Table reveals three groups of countries: (1) countries with the safe and secure economic growth: Japan, Germany and France, (2) countries with insecure and unsustainable economic growth: China, Brazil and India and (3) the borderline countries where economic growth might become unsustainable: USA, UK, Canada and Australia.

4.1. Japan, Germany and France

These three countries could serve as an examples of a prudent and secure economic growth. Maybe poorer countries can have a good reason to increase substantially and quickly their economic output because their per capita output is low but this rationale does not apply to richer countries. Their GDP/cap is already high and there is no urgent need to make it substantially higher. Corrections could be made for the increasing population in these countries but these corrections would have to be small.

Economic growth in Japan, Germany and France is safe and sustainable. Their economic stress factors in 2015 were exceptionally low and the projected stress factors are also relatively low.

However, there is a serious warning for these countries. They approach asymptotic maximum and their economic growth rates are now around 1%. When the growth trajectory is approaching an asymptotic maximum, it is hard to control its growth. Ideally, the growth rate should be also decreasing asymptotically to zero but in practice it is probably impossible to do it. These countries should not be tempted now to increase substantially their economic growth rates; otherwise the mistake made in Greece might be repeated and the safe growth trajectory might be easily diverted to a runaway process. The growth rate, in these countries should be kept below 1% and should not be constant for a long time because positive constant growth rate, even if small, generates exponential growth, which over sufficiently long time is unsustainable. There should be no alarm if the growth rate fluctuates around zero value. Under these conditions, economic growth in these three countries will remain safe and secure.

Table 3. *Summary of predicted economic growth*

Country	*GDP (2000)*	*GDP (2015)*	σ *(2015)*	*GDP (2030)*	σ *(2030)*	$\Delta\sigma$ *(2015-2030)*	*GDP (2045)*	σ *(2045)*	$\Delta\sigma$ *(2030-2045)*	σ *(t_{max})*
USA	12.713	15.597	1.23	21.418	1.68	37%	24.731	1.95	15%	2.04
China[a]	2.237	8.909	3.98	37.998	16.99	327%	157.992	70.63	316%	NPM
	2.237	8.909	3.98	29.341	13.12	229%	82.044	36.68	180%	570
Japan	5.348	5.986	1.12	6.455	1.21	8%	6.539	1.22	1%	1.23
Germany	3.120	3.697	1.18	4.209	1.35	14%	4.544	1.46	8%	1.60
France	2.347	2.775	1.18	3.06	1.30	10%	3.21	1.37	5%	1.43
UK	2.076	2.683	1.29	3.474	1.67	29%	4.062	1.96	17%	2.60
Brazil[b]	1.560	2.319	1.49	4.611	2.96	99%	8.902	5.71	93%	NPM
	1.560	2.319	1.49	5.011	3.21	116%	17.63	11.30	252%	I
India[c]	0.812	2.295	2.83	8.326	10.25	263%	37.543	46.24	351%	NPM
Canada	1.343	1.793	1.34	2.339	1.74	30%	2.936	2.19	26%	NPM
Australia	0.846	1.301	1.54	1.877	2.22	44%	2.545	3.01	36%	5.52

Notes: σ (t_{max}) – σ at the time of predicted maximum. NPM – No predictable maximum because of the ever-increasing trajectory. I – Infinity. [a]) The first row is for the exponential trajectory as in the past few decades. The second row is for the second-order exponential trajectory as suggested by the recently decreasing growth rate. [b]) The first row is for the second-order exponential trajectory. The second row is for the pseudo-hyperbolic trajectory. [c]) These values are for the second-order exponential trajectory. If economic growth in India is going to follow the pseudo-hyperbolic trajectory, the growth will collapse before 2029. Note: $\Delta\sigma(2015-2030)$ were calculated using empirical values for $\sigma(2015)$. $\Delta\sigma(2030-2045)$ were calculated using projected values.

4.2. China, Brazil and India

Economic growth in China was following an excessively fast exponential trajectory. In order to maintain this fast growth, economic stress factor would have to increase to $\sigma = 16.99$ in 2030 and to $\sigma = 70.63$ in 2045, corresponding to the 327% and 316% increase, respectively, in the two consecutive 15-year intervals. It is doubtful that such enormous increase over such a small time can be achievable. To continue this fast growth in the future, economic output of one year in 2000 would have to be generated in 21 days in 2030 and in 5 days in 2045.

There is, however, indication that the growth rate in China started to decrease, but the corresponding generated trajectory is still too fast. Its stress factors are still substantially large: $\sigma = 13.12$ in 2030 and to $\sigma = 36.68$ in 2045, corresponding to the to 229% and 180% increase, respectively, in the two consecutive 15-year intervals. In order to support this slower economic growth, economic output of one year in 2000 would have to be generated in 29 days in 2030 and in 10 days in

2045.A safer way for China would be to start reducing the growth rate suitably fast to achieve a secure economic growth.

Economic growth in Brazil and India is potentially catastrophic because their growth rates are not only high but also increasing. Even a constant growth rate is dangerous because it generates an unsustainable growth trajectory, but the increasing growth rate is even worse.

There are two possible future trajectories for Brazil: (1) a fast-increasing second-order exponential trajectory and (2) an even faster increasing pseudo hyperbolic trajectory. They describe excessively fast economic growth, which because of its rapid increase can be easily unmanageable. In addition, if the economic growth in Brazil is going to follow the pseudo-hyperbolic trajectory, it will inevitably lead to the economic collapse before 2053 because this trajectory increases to infinity in that year.

Economic growth in India is in the same category as in Brazil. Economic growth rate is also increasing. If this pattern is going to continue, it will result in an economic collapse.If the growth is going to follow the pseudo-hyperbolic growth, then the economic growth in India will collapse before 2029 because their trajectory escapes to infinity in that year. It could be something similar to the past experience in Greece. Their economic growth trajectory was escaping to infinity in 2017 and their economic growth collapsed around 2008. If the economic growth in India is going to increase along a slower, but still fast-increasing second order exponential trajectory, economic collapse will be only delayed but not avoided. Economic growth rate in Brazil and India should be now decreasing sufficiently fast to create a more secure future. The road to prosperity will be slower but safer.

4.3. USA, UK, Canada and Australia

Economic growth in the USA, UK, Canada and Australia is potentially insecure. Parameters listed in Table 3 indicate that economic growth in these countries might continue undisturbed because the increase in the economic stress in the two consecutive 15-year intervals is not excessively large. However, to sustain their growth trajectories, economic output of 2000 would have to be generated in about 6 months in 2045 in the US, UK and Canada and in about 4 months in Australia. At the time of the projected maximum, economic output of one year in 2000 would have to be generated every 5 months in the UK and every 2 months in Australia.

Ideally, these countries should experience only small increase in their economic stress factors, as in Japan, Germany and France, and there is no good reason for such considerable disparities between these two groups of rich countries because economic output per person in all of them is already high. A safer approach would be to start reducing their growth rates sufficiently fast to avoid overheating.

4.4. Concluding remarks

Economic growth in the leading economies presents mixed fortunes. Only three countries have a safe and secure economic growth but their future depends on how successfully they can control their growth rate, which is now very low. Even these countries are in danger if they allow for their growth rate to increase consistently. For other countries, economic growth rate should be decreasing sufficiently fast to create a stable and secure economic future. The GDP will still continue to increase but it will increase in a safe and sustainable way.

The general aim now should be to reduce the growth rate eventually to zero. Growth rate can be allowed to increase or stay constant only for a very limited time. The way the growth rate should be, if possible, reduced depends on the economic status of a given country. If the growth rate is reduced too fast and if the initial economic status of a given country is low, the resulting GDP, when the

growth rate is going to approach the zero value might be too low to support tolerable standard of living. If the economic status of a given country is already high, the reduction of the grow th rate to zero could be achieved much safer and would ensure a sustainable economic future. The general drive to maximise the economic growth rate and to keep it high is a serious mistake.

Appendix

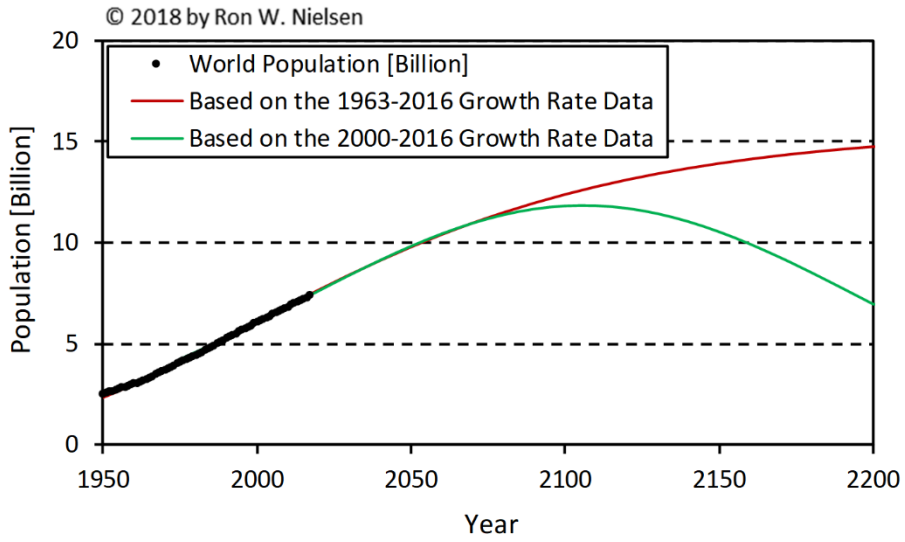

Figure 1. *Predicted growth of the world population. The predicted growth is in excellent agreement with the predictions published by the United Nations (2015). For a long time into the future, the two projected trajectories will be identical. The difference between the two projections will become apparent only close to the end of the current century or even at the beginning of the next century.*

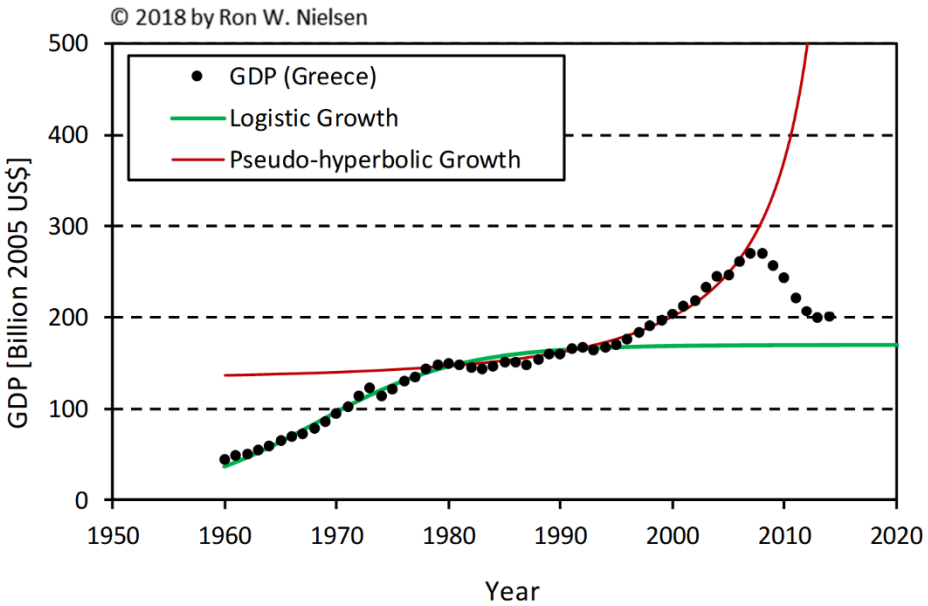

Figure 2. *Economic crisis in Greece was predictable. After following a logistic trajectory characterized by the fast-decreasing growth rate, economic growth was diverted to a fast, pseudo-hyperbolic trajectory with singularity in 2017, when the GDP would have to increase to infinity, which was impossible. Under these conditions, economic collapse was inevitable, but it could have been prevented. The recommended option for Greece is to start to increase the growth rate, if possible, but slowly to avoid the danger of the earlier runaway process. After reaching a satisfactory level of the GDP, the growth rate should start to decrease slowly.*

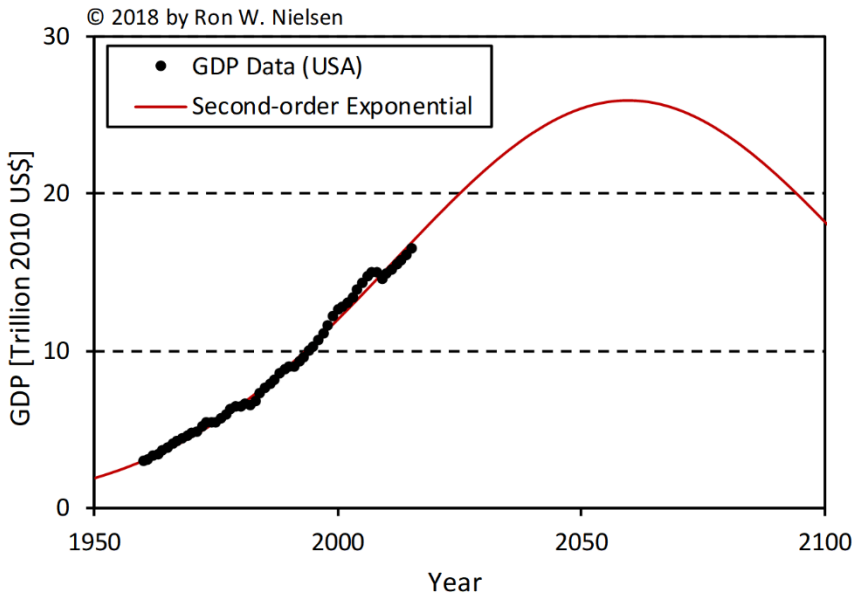

Figure 3. *Economic growth in the US is represented by the second-order exponential trajectory leading to a maximum in 2060. It is a potentially sustainable growth. The predicted economic stress factor in 2060 is* $\sigma = 2.04$. *The recommended option for the US is to keep the decreasing growth rate at least at the same rate as in the past few decades but a faster decrease would be more secure.*

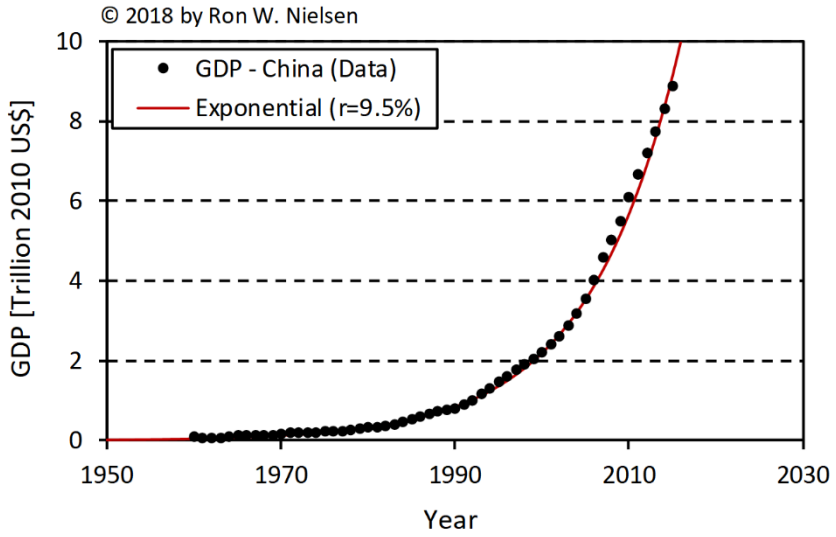

Figure 4. *The past economic growth in China was approximately exponential with the average growth rate of around 9.5% per year, which corresponds to the doubling time of 7.3 years. The GDP increased from around $2.24 trillion (of 2010 US$) in the year 2000 to around $8.91 trillion in 2015, i.e. by a factor of around 4.*

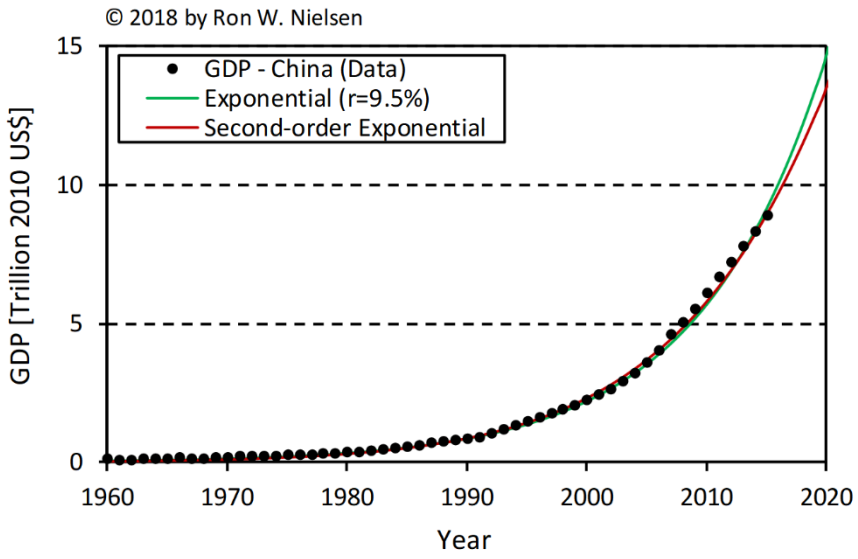

Figure 5. *Comparing two growth trajectories for China. The exponential growth is based on the examination of the full range of growth-rate data. However, from around 1980, the growth rate appears to be slowly decreasing. If this trend is used to calculate growth trajectory, it generates the second-order exponential growth, which leads to a maximum (see Figure 6).*

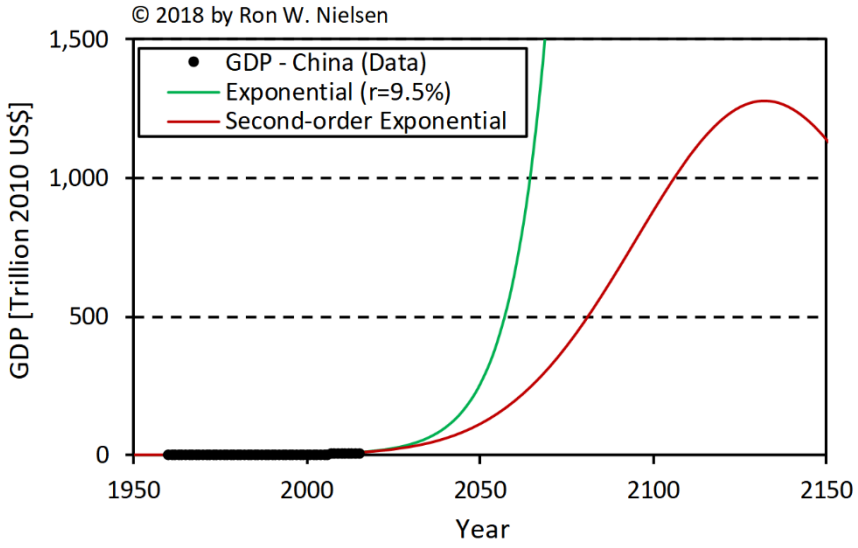

Figure 6. *If the growth rate is going to continue to decrease, as suggested by the recent growth rate data, economic growth in China will not be increasing exponentially, as in the past few decades, but will follow a second-order exponential trajectory leading to a maximum. This trajectory is still too fast. The recommended option for China is to start to decrease their economic growth rate sufficiently fast to divert their economic growth to a slower but sustainable trajectory.*

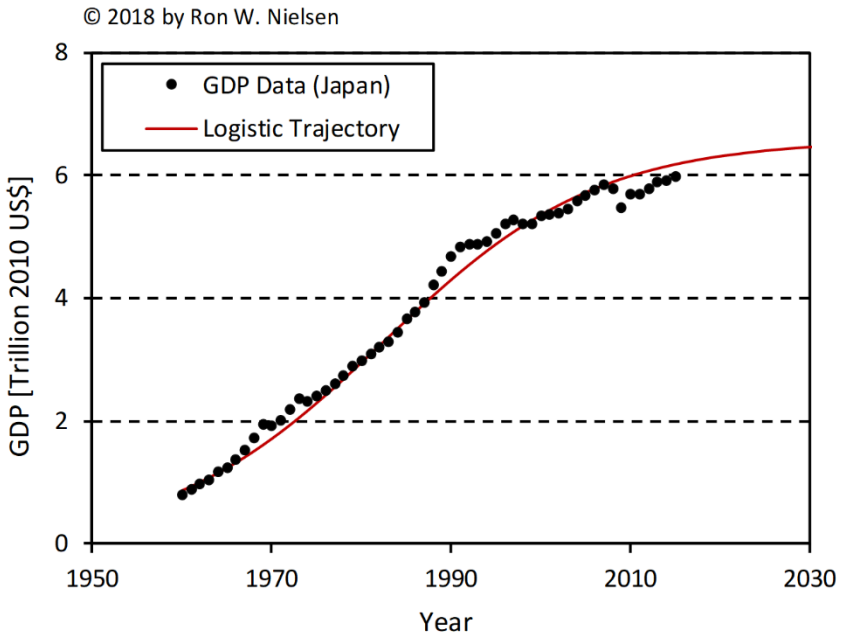

Figure 7. *The most secure economic growth in the group of countries listed in Table 1 is in Japan. The displayed curve was not calculated by fitting the GDP data but by the analysis of the economic growth rate. It is, therefore, a good test of the presented here mathematical method of predicting growth. The best recommended option for Japan is to maintain the economic growth rate, if possible, close to zero. Any attempt to increase the growth rate and to keep it high should be avoided.*

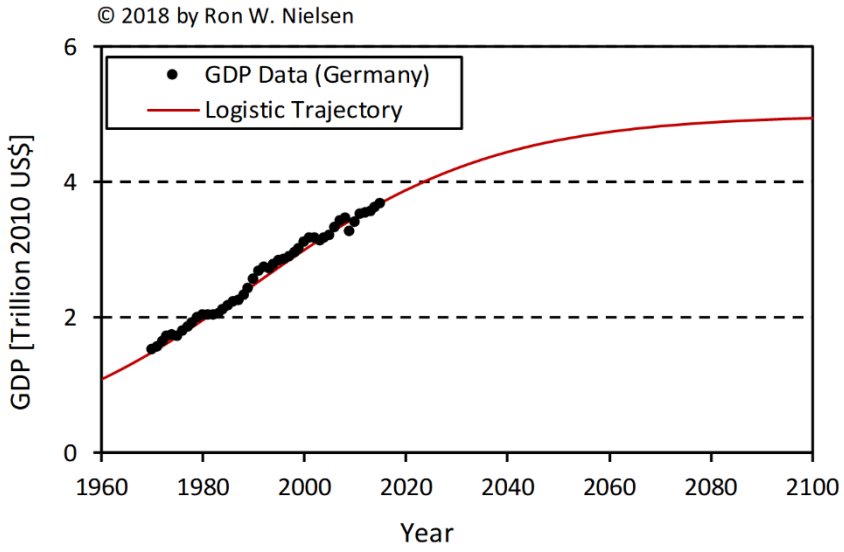

Figure 8. *Economic growth in Germany follows a secure logistic trajectory. Like Japan, Germany is also now at the stage when the growth rate is low and should be maintained, if possible, at its small value, decreasing gradually to zero. Any attempt to increase the growth rate and to keep it high should be avoided.*

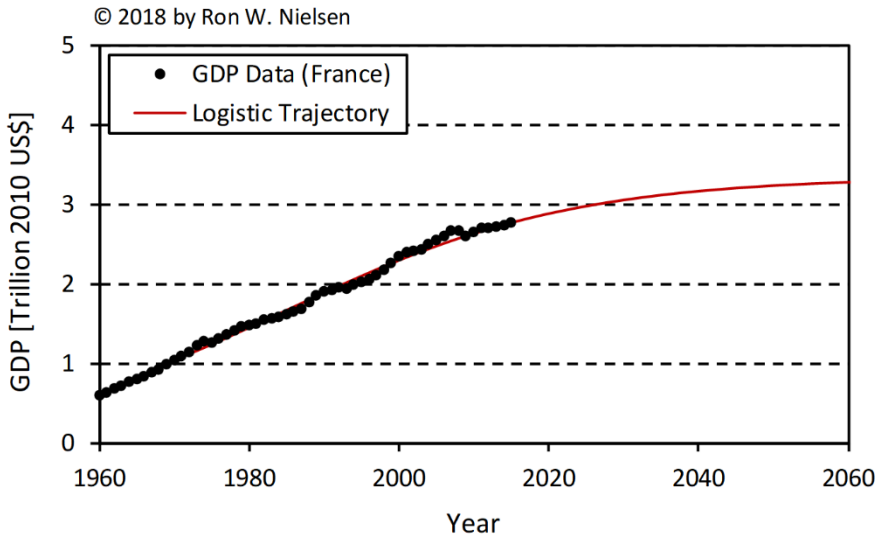

Figure 9. *As in Japan and Germany, economic growth in France is now also at the stage when the growth rate is low and it should be kept not only low but gradually decreasing to zero to approach the safe asymptotic maximum of the GDP. Any attempt to increase substantially the growth rate should be avoided.*

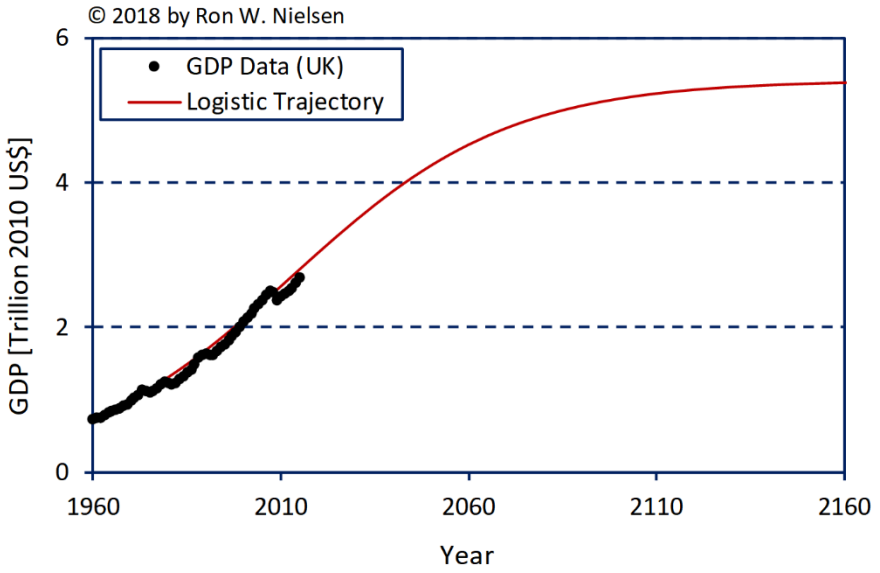

Figure 10. *Economic growth in the United Kingdom follows a logistic trajectory. This growth might be still sustainable during the first half of the 21st century but might become unsustainable in the second half because of a considerable economic stress when approaching the asymptotic value of the GDP. To ensure a sustainable growth, economic growth rate should start to be reduced faster than in recent years.*

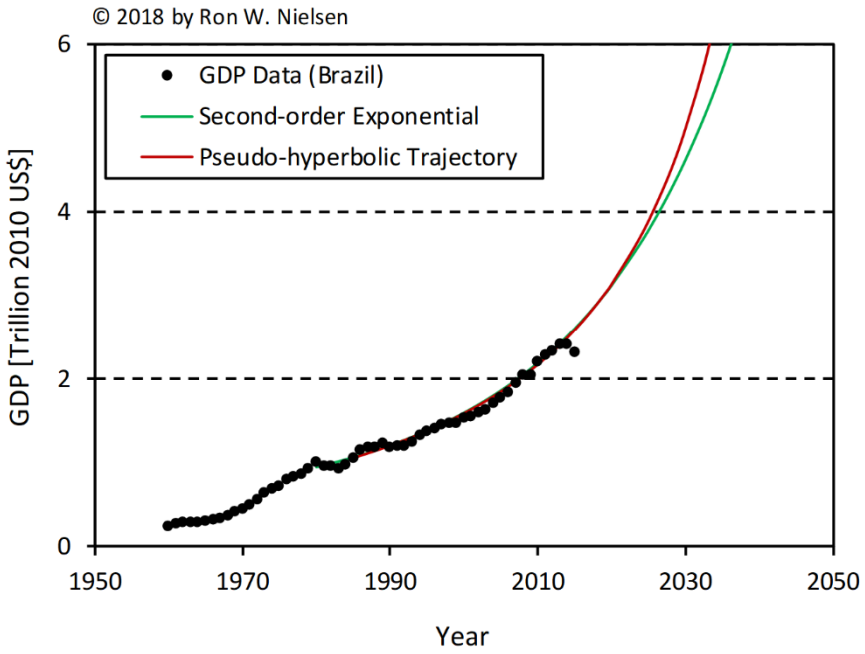

Figure 11. *From around 1985, growth rate describing economic growth in Brazil was increasing and generating unsustainable economic growth. The corresponding trajectories are (1) the ever-increasing second-order exponential trajectory and (2) the pseudo-hyperbolic trajectory, escaping to infinity in 2053. Assuming that the recent decline in the economic growth is just a temporary aberration, the ever-increasing growth will be unsustainable. Brazil should now start to decrease their growth rate but to keep it positive.*

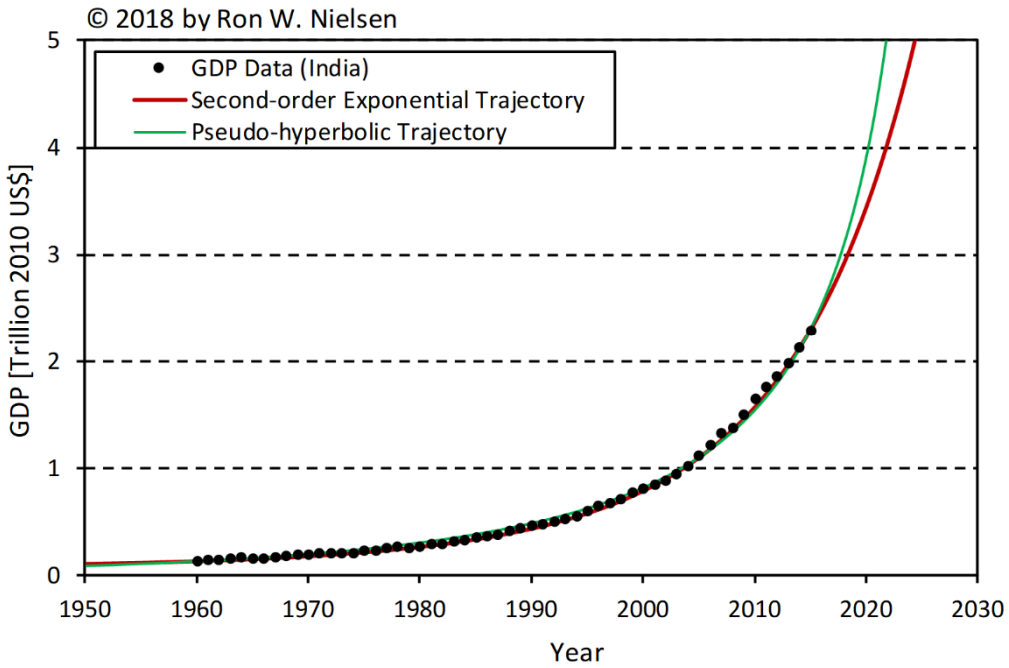

Figure 12. *Economic growth in India is critically unsustainable. One of the two possible patterns of growth is described by the pseudo-hyperbolic trajectory, which escapes to infinity in 2029. However, even a little slower, second-order exponential growth, is also critically unsustainable. India should now start to decrease the economic growth rate. On no account, the growth rate should be allowed or prompted to increase.*

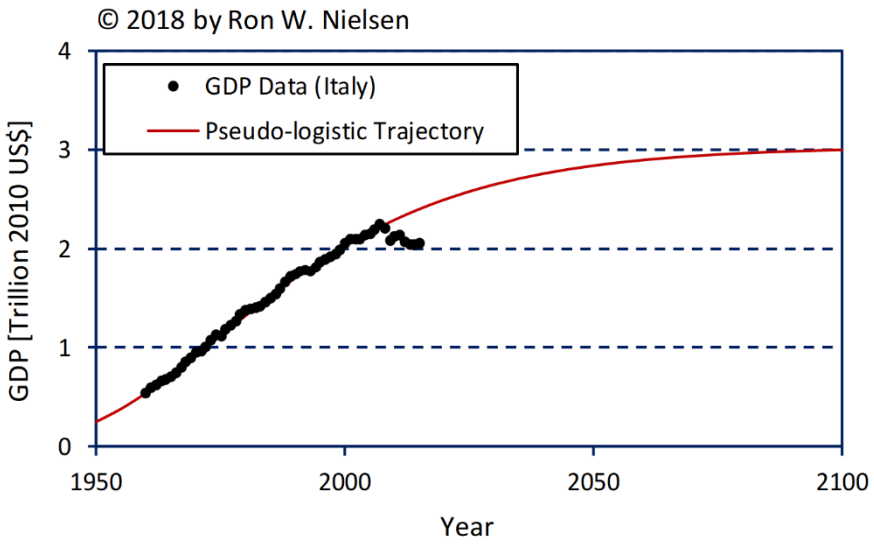

Figure 13. *Economic growth in Italy was increasing steadily by following a pseudo-logistic trajectory but it unexpectedly collapsed in around 2008. Future growth is unpredictable.*

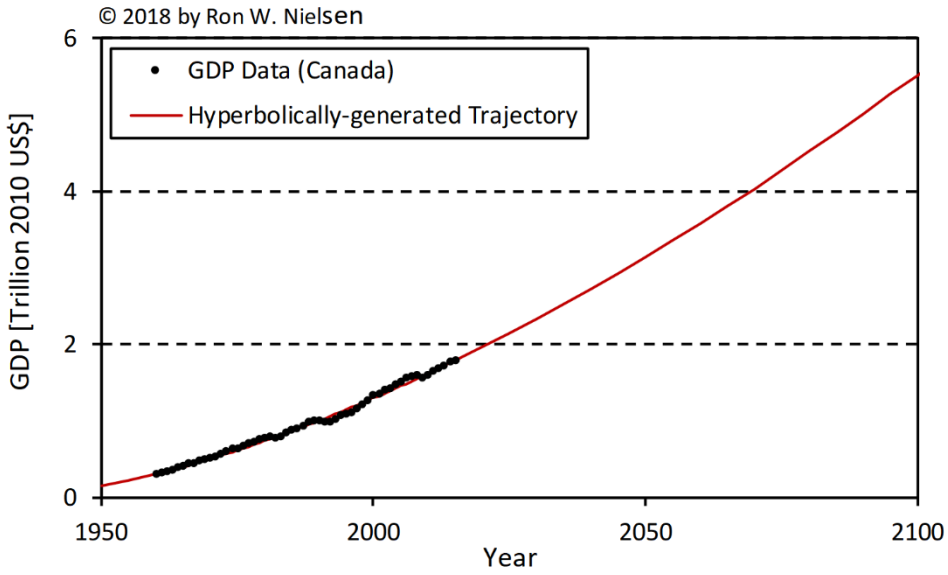

Figure 14. *Economic growth in Canada has been following the ever-increasing trajectory since 1960. Over a sufficiently long time, such a continually-increasing trajectory is unsustainable but it might be still tolerated in the near future because the expected increase in the economic stress factor during the current century is not excessively high. However, it would be safer for Canada to start to decrease the economic growth rate faster than in the recent past.*

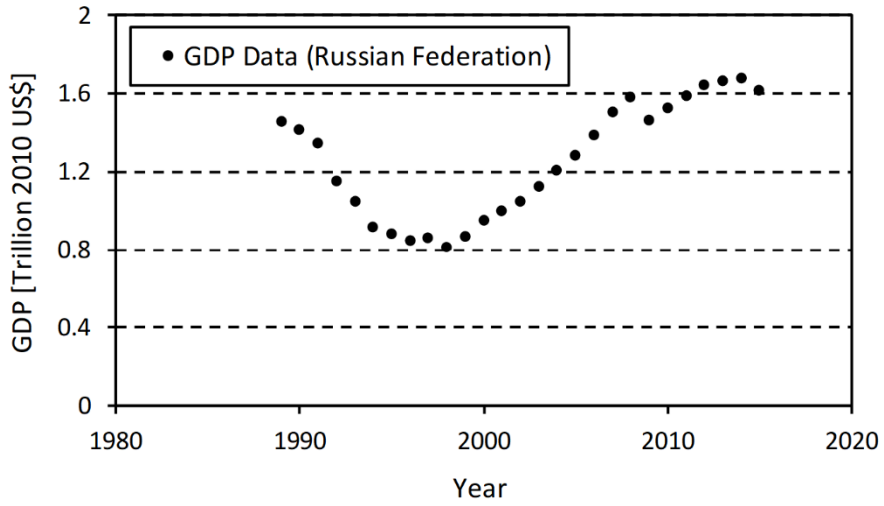

Figure 15. *Economic growth in Russian Federation is strongly unstable and unpredictable.*

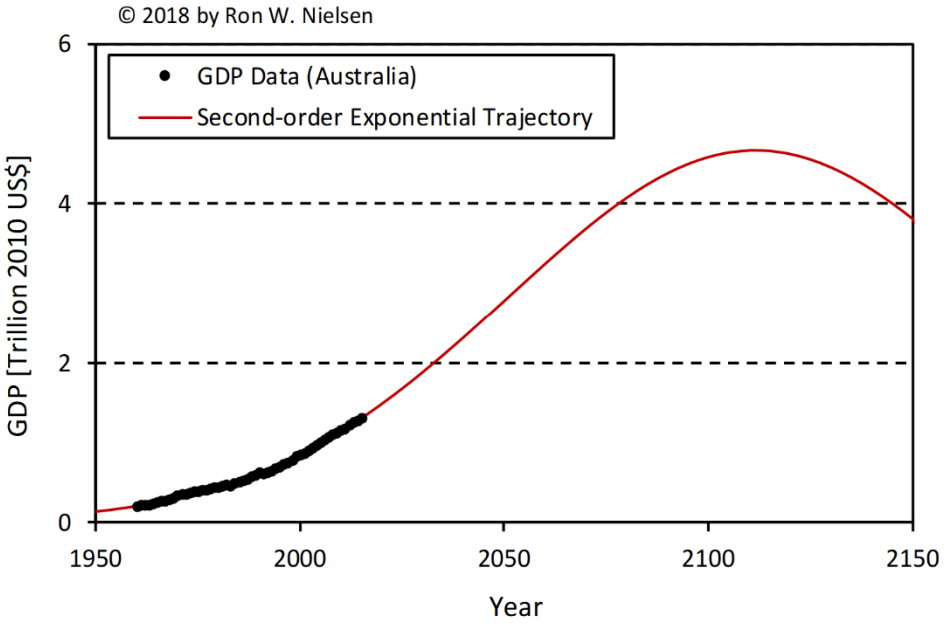

Figure 16. *Economic growth in Australia can be described by the second-order exponential trajectory with maximum of $4.472 trillion (2010 US$) in 2111 corresponding to* $\sigma = 5.52$ *This is a fast-increasing trajectory and potentially unsustainable.*

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